The Window Theory

Physics as the topology of causality

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Abstract

We start from one postulate—an effect cannot be the cause of itself—and derive its consequences. The result is a directed acyclic graph whose causal edges carry compact U(1) phases. Requiring local gauge dynamics to close on the minimal complete causal window selects a unique graph: $\binom{5}{2} = \binom{5}{3} = 10$, hence the complete graph K_5.

K_5 is not a spacetime brick; it is a sliding causal window. Time is continuation; space is synchronisation. Particles are stable causal motifs: charged leptons are phase defects, baryons are k=3 closures, pions are k=2 bridges, photons are coherent phase transport, the Higgs is the radial stiffness of causal ordering.

From this single object we derive, with no free parameters: the SM gauge group $\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)$; the fermion spectrum $\bar{5} \oplus 10$ in three generations; the bare orientation coupling $\alpha^{-1}_{\mathrm{bare}} = |A_5| = 60$, dressed by charged-motif loops to $\alpha^{-1}_{\mathrm{IR}} \simeq 137.04$; $\sin^2\theta_W = 0.232$; the causal depth $L^* = 26$; the Higgs ratio $m_H/v = 3/\sqrt{35}$ (124.9 GeV, 0.2%); the proton radius law $m_p r_p/(\hbar c) = 4$ (0.04%); $m_\pi/m_p = 1/(3\sqrt{5})$ (0.2%); the neutrino law $\Delta m^2_{21}/\Delta m^2_{32} = 1/35$; the gravitational response $V_0 = 1/(10\pi)$; the operator-level vacuum split $\Omega_\Lambda = 2/3$; and the dark-matter frontier relic candidate ratio $\Omega_{\mathrm{DM}}/\Omega_b \simeq 5.48$ (0.4%).

Scattering is a boundary-to-boundary response kernel on the thermodynamically selected causal skeleton; unitarity is reconstructed in the infrared stable-motif sector; Lorentz invariance emerges as the kinematics of the surviving gapless transport sector. All claims are classified as theorem, derived law, candidate, benchmark, or open structural target.

How to read this manuscript. The text is intentionally bold, but it is not meant to blur what is proven and what remains open. A claim labelled **THEOREM** is an exact consequence of the K_5 structure or its algebra. A **DERIVED LAW** is obtained from already established K_5 objects and normalisations. A **CANDIDATE** is a mechanism with a causal chain and numerical support but with at least one operator-level step not yet closed. An **OPEN** problem is explicitly not presented as solved.

This first part is not a popular replacement for the technical text. It is a causal reading guide. Each radical claim is stated in ordinary language and then linked back to the same chain:

$$\text{causal order} \to \text{edge phase} \to K_5 \text{ window} \to \text{gluing} \to \text{stable motif} \to \text{observed effect.}$$

How to read a radical claim. A radical claim in this manuscript is never meant to stand alone as philosophy; it is to be read as a causal chain. For example, the statement that the Higgs resonance is "not a fundamental matter particle" is not a rhetorical statement about names. It means that the object identified experimentally as a 125 GeV scalar is represented mathematically in this theory as the radial ordering mode of the causal window ($A_5 \to A_4$), whereas charged fermion masses are computed separately from topological defect costs. The claim is ontological, not observational: the resonance is real, but the primitive explanation is different.

The same discipline applies to Bell correlations, quarks, virtual particles, and dark matter. The manuscript does not deny the experiments. It changes the object to which the experimental language is attached. A useful translation rule: *observable phenomenon → usual ontology of that phenomenon*. The theory keeps the former and re-derives the latter from causal topology.

Contents

Part I

The Causal Picture

1 Starting before space and particles

Physics often begins with space, time, particles, and fields. This framework begins earlier. It assumes only causal order: an effect cannot be the cause of itself. Thus events form a directed acyclic graph. At this point there is no background space, no particle inventory, and no force law. There are only events and causal arrows.

If two events are causally connected, they cannot be identical. The connection must carry a distinction. The minimal distinction on one causal edge is a phase. A single phase on a single edge is not itself an observable; physical content appears when phases fail or succeed to agree around closed comparisons. This is the origin of gauge content in the theory.

Local dynamics for these phases must be gauge-invariant, local, and positive. The natural local action is therefore Wilson-type:

$$S = \beta \sum_f (1 - \cos \Phi_f), \tag{1}$$

where Φ_f is the phase mismatch around a minimal face. Now one asks for the smallest local window in which the number of edge degrees of freedom equals the number of face equations. For a complete graph on $d+1$ vertices this condition is

$$\binom{d+1}{2} = \binom{d+1}{3}. \tag{2}$$

The nontrivial solution is $d = 4$, hence the minimal closed causal-gauge window is K_5.

The K_5 window is not chosen as a convenient lattice cell. It is the smallest complete causal window in which edge phases and face constraints close on one another.

The K_5 window is also not a brick of pre-existing space. It is a sliding window of a causal stream. Five successive ticks form one window; the next tick shifts the window by one step. Time in this theory is therefore not first a coordinate. Time is the operation of continuation. Extended space appears later, when many such streams synchronise through compatible windows.

The choice of K_5 is not merely a geometric convenience. The abstract symmetry code of the five-vertex window (A_5, 60 elements) is the same code that produces icosahedral shells in chemistry and biology. In an ordinary spatial system, where all directions are reversible, this code naturally forms closed finite structures. But when constrained by a causal arrow, the same code cannot close without violating the prohibition on causal loops. Thus the very principle that creates stable, finite motifs in space is forced to become continued growth in time. Expansion is the causal projection of the same closure principle that produces matter.

2 What exists: causal motifs, not particle bricks

The theory does not first postulate particles. It classifies stable causal motifs of the K_5 window. A motif may be a local defect, a bridge, a closure, a neutral carrier, a radial ordering mode, or a collective network mode. Ordinary particle language appears only after such motifs have been identified.

This changes the status of familiar objects. An electron is not the same kind of object as a proton. A proton is not three little balls. A pion is not a stable primitive object in the same sense as a proton. The Higgs resonance is real, but it is not a fundamental matter particle. A photon is not a light-ball moving through a pre-existing space. These are different motifs, with different stability conditions.

3 Photon as phase transport

In this framework a photon is the stable propagation of U(1) phase along a causal stream. The phase lives on causal edges. When the K_5 window slides, part of the phase structure is carried across the shared K_4 boundary. If that phase transport is coherent over successive windows, the result is a stable massless transport motif. This is what the photon is in K_5 language.

photon = coherent U(1) *phase transport through sliding* K_5 *windows.*

Frequency is the rate of phase rotation per causal continuation step. Polarisation is the way that phase transport is distributed across the spatial channels of the window. Nothing here requires a small object flying through an already-given space. Space is the synchronised large-scale organisation of the same windows.

Status: Causal interpretation. The technical gauge sector and edge-phase construction are part of the main body.

4 Bell correlations without magical transmission

Bell-type experiments are often presented as a puzzle about two distant particles. The particles separate. Later one chooses a measurement on the left and a measurement on the right. The outcomes are correlated more strongly than a classical local hidden-answer picture allows. It is then tempting to say that one particle somehow tells the other what happened.

The K_5 view rejects the first sentence of that story. The primary object is not two independent particles. The primary object is one causal motif with a common phase structure and common gluing constraints. What is later described as two distant systems are two local projections, or two distant boundaries, of that earlier motif.

The usual local hidden-variable factorisation assumes that, once some hidden state λ is given, the left and right responses are independent:

$$P(A,B|x,y) = \sum_{\lambda} \rho(\lambda)\, P(A|x,\lambda)\, P(B|y,\lambda). \tag{3}$$

In words: each side is supposed to carry a ready answer to every possible local question, and the joint result is built by multiplying left and right local answer tables.

This is not the K_5 ontology. The hidden state is not a pair of independent little objects. It is one causal motif. A measurement setting is a choice of local projection of that motif. Two incompatible local projections do not have to coexist as a pre-written table of facts. Therefore the compatibility of two local outcomes need not factor into a left answer multiplied by a right answer. A more faithful causal form is

$$P(A,B|x,y) = \sum_{\lambda} \rho(\lambda)\, C_{\lambda}(A,B;x,y), \tag{4}$$

where C_λ is a compatibility condition for two local projections of one common motif, not a product of two independent tables.

There is no new superluminal causal arrow from left to right. The correlation is not transmitted at the moment of measurement. It is inherited from the earlier common motif and from the fact that the two later outcomes must be compatible projections of that motif.

Bell correlations do not force magical transmission. They force the rejection of the particle-first assumption that two separated systems already carry complete independent answer tables.

Einstein was right to reject mysterious action at a distance. The K_5 theory rejects it as well. But the classical alternative is also wrong: the world is not a set of independent little objects equipped with all possible answers. The primary object is the causal motif, and measurement selects a local projection of it.

Status: Causal-topological interpretation. The full derivation of the Born rule, correlations, and decoherence belongs to the technical quantum-sector sections.

5 Higgs: a real resonance, not fundamental matter

The common slogan says that the Higgs gives mass to particles. In the Standard Model as a parameterisation this is useful, but it does not explain the fermion masses: the Yukawa constants are free inputs.

In K_5 the fermion masses are not put in through Yukawa constants. They arise as causal costs of stable motifs: defects, chain screenings, and closures. The electron, muon and tau hierarchy is controlled by local curvature and causal-depth screening. The proton mass is controlled by a closure action. Thus the Higgs is not needed as the origin of fermion masses.

What, then, is the Higgs? The K_5 window has an A_5 symmetry. The causal ordering selects an A_4 stabiliser. The four-dimensional representation decomposes as $4 \to 1 \oplus 3$. The three directions are Goldstone-like directions of the broken orientation. The remaining singlet is the radial stiffness of the ordered phase. The observed 125 GeV resonance is identified with this radial mode.

$$\textit{Higgs} = \textit{radial stiffness of the causal ordering } A_5 \to A_4.$$

The resonance is real. Its ontological status is different: it is not fundamental matter, but a collective ordering mode of the causal window.

Status: Derived law for m_H/v in the technical body; interpretation as radial ordering mode is central to the K_5 ontology.

6 Quarks and baryons

A coloured object is not a complete stable causal motif. It is a channel of a larger motif. The proton is not three independent little constituents arranged in space. In K_5 it is the minimal colour-neutral k=3 closure. What ordinary language calls a quark is a coloured channel inside a closure description.

$$\textit{quark} = \textit{coloured channel of an unfinished closure motif.}$$

This is why the theory should not seek a free quark mass as if it were the mass of an isolated object. The stable physical object is the closure. The colour channels are meaningful inside the closure, not as standalone causal objects.

Status: Closure-sector spectra and baryonic observables are technical results; the no-standalone-quark interpretation follows from the closure ontology.

7 The new primitive $L^* = 26$

The number 26 is not a spatial length and not a number of cells in the universe. It is the full non-point relational content of one K_5 window:

$$L^* = \dim \Omega^{\geq 1}(K_5) = 10 + 10 + 5 + 1 = 26. \tag{5}$$

The terms are edges, faces, tetrahedra, and the top form. Vertices are point events; they do not carry relational content by themselves.

$$L^* = 26 \textit{ is a new kind of physical primitive: causal relational depth.}$$

It controls chain screening and appears in the charged-lepton hierarchy. It is not fitted. It is the number of non-point relations a K_5 window can carry.

Status: THEOREM in the technical body.

8 Virtual particles

In perturbative quantum field language, a virtual particle is an internal line of a diagram. It is not observed as an incoming or outgoing particle and need not satisfy an on-shell mass relation. Even in that language it is not a particle in the same sense as a detected electron or photon.

In K_5 the distinction becomes ontological. The fundamental object is a causal window with phases on edges. On a finite region the theory sums over admissible phase configurations:

$$Z = \int_{\mathrm{U}(1)^E} \prod_e d\theta_e \ \exp\bigl(-\beta \sum_f (1 - \cos \Phi_f)\bigr). \tag{6}$$

There are no little internal objects flying between events inside this expression. There are phase configurations, gluing constraints, and response kernels.

A real observed particle is a stable causal motif: a defect, bridge, closure, carrier, radial ordering mode, or collective transport mode. A virtual particle does not pass this stability test. It is a contribution of unstable or off-saddle phase configurations to the response of the network.

For example, when two charged defects interact, the K_5 description is not that a small virtual photon flies between them. The interaction is the correlated response of the U(1) phase sector between two charged motifs. In perturbative language this response can be drawn as an internal photon line. In K_5 language it is a phase-response kernel.

virtual photon = shorthand for a U(1) *phase-response kernel, not a flying object.*

The same reading applies to loop effects. They are corrections from nonlinear and anharmonic parts of the phase action around an ordered configuration. Diagrams may be convenient, but they are not the ontology.

9 Dark energy and dark matter

Dark energy is not introduced as a new field. In the K_5 collective sector the $\mathbb{Z}_3$ decomposition gives a matter/vacuum split. The internal vacuum fraction is

$$\Omega_\Lambda^{\mathrm{vac}} = \frac{2}{3}, \tag{7}$$

and the corresponding coefficient in

$$\rho_\Lambda = \frac{H_0^2\, M_{\mathrm{Pl}}^2}{4\pi} \tag{8}$$

is the same statement in standard cosmological normalisation.

Dark matter is different. The minimal theory contains no hidden particle dark sector. The only surviving K_5 mechanism is a primordial frontier relic: an excess of nonluminous causal structure seeded by baryonic closures during early network growth and then frozen into the mature network. This mechanism is a candidate, not yet a theorem, because the final freeze-out operator remains to be closed.

Status: Dark energy: derived/operator-level in the technical body. Dark matter: candidate frontier-relic mechanism.

10 What this theory changes

The theory does not merely add a field or a parameter. It changes the order of explanation.

- Space is not first; causal continuation is first.
- Particles are not first; stable causal motifs are first.

- The Higgs resonance is real, but it is not fundamental matter.
- Quarks are coloured channels of closure motifs, not standalone objects.
- Bell correlations do not require action at a distance; they reveal the failure of independent answer tables.
- Virtual particles are not objects of the world; they are terms in a response expansion.
- Dark energy is a collective vacuum-sector split, not an added field.
- Particle dark matter is absent in the minimal theory; the only current K_5 route is a primordial frontier relic.
- The number $L^* = 26$ is a new physical primitive: relational depth.
- Scattering cross sections are boundary-to-boundary response kernels of the phase network, not exchanges of virtual objects. The Rutherford, Mott, and Møller results are recovered structurally.

11 What would falsify the minimal theory

A radical theory should expose itself to clear failure modes. The minimal K_5 version would be put under direct pressure by any of the following:

- a fourth Standard-Model-like generation;
- a stable fundamental particle dark-matter sector;
- proton decay through heavy X, Y-type gauge bosons;
- neutrinos forced to be Dirac particles by experiment;
- tree-level flavour-changing neutral currents not accounted for by the K_5 hierarchy;
- a failure of the dimensionless closure laws, such as $m_p r_p/(\hbar c) = 4$ or $m_\pi/m_p = 1/(3\sqrt{5})$;
- a failure of the radial-ordering relation $m_H/v = 3/\sqrt{35}$.

The theory is therefore not protected by adjustable parameters. Its strength is also its risk: many observables are tied to the same causal window.

12 Relation to adjacent programmes

The present construction is adjacent to several research programmes that also begin from discrete or causal structures: causal sets [19, 20, 21], causal dynamical triangulations [22, 23], spin foams and loop quantum gravity [24, 25], Regge calculus [5], lattice gauge theory [1, 34], and deterministic/cellular-automaton approaches to quantum mechanics [26].

It differs from all of them in one structural point: the local building block is not a generic simplex, a generic graph-update rule, or a generic spin network. It is the unique self-contained causal-gauge window K_5, selected by the edge/face closure condition $\binom{5}{2} = \binom{5}{3} = 10$ and extended by sliding continuation. This uniqueness is the origin of the predictive power claimed in the present work: the entire particle spectrum, gauge structure, and cosmological parameters follow from one object, not from a class of models.

The closest relative is the causal set programme [19], which shares the starting point (causality as the sole primitive) but does not impose phases on edges, does not have a gauge structure, and does not select a unique graph. Lattice gauge theory [1] shares the Wilson action but starts from a regular lattice in a pre-existing spacetime, whereas K_5 derives both the lattice and the spacetime from a single postulate.

13 What is deliberately not claimed

A clean manuscript must be as precise about what it does not claim as about what it claims.

The minimal theory does not claim that every numerical observation has been derived from pure algebra. It does not claim that the Hubble tension is definitively solved. It does not claim that the dark-matter frontier relic is already a closed theorem. It does not claim that every nuclear binding energy is known from a closed K_5 formula.

What it does claim is that many of the dimensionless structures normally treated as independent—mass hierarchies, couplings, collective responses, shell closures—are shadows of one causal window. The remaining gaps are localised open operator problems inside the K_5 network, not hidden free parameters.

Part II
Technical Derivation

14 From Postulate to K_5

14.1 The postulate

Causality. *An effect cannot be the cause of itself.* This is the sole postulate. It means: events form a directed acyclic graph (DAG)—there are arrows, but no loops.

From this postulate the following chain is forced: edge phase → Wilson action → closure at $d = 4$ → K_5 as a sliding window → 3+1 from window structure → ordered phase, homogeneous in time. Every step is forced. From this ordered phase, all remaining physics follows: gauge group, chirality, three generations, the Higgs boson, gravity, and the expansion of the Universe.

14.2 The causal graph

Reality is modelled as a set of events and causal links—a directed acyclic graph (DAG): following the arrows, one cannot close a loop. From this alone: space is the graph distance (number of steps); time is the direction of the arrows; the speed limit is one step per step (the speed of light is not a postulate but a consequence of discreteness). Neither space, nor particles, nor forces are postulated.

14.3 Edge phase

Two events are connected by an edge. The link cannot be null—otherwise the events are indistinguishable and constitute a single event. Each edge carries a nonzero datum—a phase θ. The phase is compact (lives on the circle U(1)); otherwise the mismatch grows without bound and the structure is destroyed.

The gauge-invariant holonomy—the sum of phases around a closed path—measures the discrepancy between direct and roundabout comparison of two events. This discrepancy is the physical content that becomes force.

Theorem 14.1 (Edge rigidity)**.** *Let the parallel transporter on a DAG edge satisfy:* (i) *inversion* $g(t{\to}s) = g(s{\to}t)^{-1}$, (ii) *coherence (cocycle condition),* (iii) *rigidity—$g(e)$ is determined solely* *by incidence data (source, target, orientation), with no additional structure. Then* $\dim G = 1$ *and* $G \cong \mathrm{U}(1)$.

Proof. An edge carries exactly one datum (the orientation). For $\mathrm{U}(n)$ with $n > 1$, one needs $n^2 - 2$ parameters without topological support on a 1-simplex. □

Corollary 14.2. *Non-abelian gauge structure (*$\mathrm{SU}(3)$*) arises not from edges but from cells—combinatorial objects with internal structure. The hierarchy is:* $\mathrm{U}(1)$ *on edges (rank 1),* $\mathrm{SU}(3)$ *on cells (rank 2, from* $10 \to 8$ *via tracelessness).*

14.4 Wilson action and the photon as a theorem

The $\mathrm{U}(1)$ phases θ_k on edges are the discrete analogue of the electromagnetic potential A_μ. Gauge transformation: $\theta_k \to \theta_k + \lambda(\text{target}) - \lambda(\text{source})$, where $\lambda(v)$ is an arbitrary phase at vertex v. Observables are gauge-invariant combinations only (holonomies around closed loops).

The dynamics is determined by the Wilson action [1]:

$$S = \beta \sum_{\square} (1 - \cos \Phi_\square), \tag{9}$$

where $\Phi_\square$ is the holonomy (total phase around the minimal closed cycle—a plaquette). This is the unique action compatible with gauge invariance, locality, positivity, and the restriction to plaquettes of ≤ 4 edges. Any other gauge-invariant local action differs only by higher-order terms ($\cos 2\Phi, \ldots$), which are irrelevant operators in $d = 4$ and vanish in the continuum limit.

The photon as a theorem. In the continuum limit ($\beta \to \infty$), $S \to \frac{1}{4g^2} \int F_{\mu\nu} F^{\mu\nu}\, d^4x$—the Maxwell action. Its spectrum contains a massless spin-1 particle, and each of these three properties is forced.

Masslessness is protected by gauge symmetry. A mass term $m^2 A_\mu A^\mu$ breaks $\mathrm{U}(1)$ and is forbidden to all orders—perturbatively and non-perturbatively (Elitzur's theorem [3]). This is not an assumption but a consequence of $\mathrm{U}(1)$, which itself follows from the edge structure.

Spin 1 follows because A_μ is a 1-form. In 3+1 dimensions, gauge fixing leaves exactly 2 transverse polarisations—the two helicity states of a massless spin-1 particle (Wigner classification [4]).

Charge quantisation follows from compactness. The phase $\theta \in [0, 2\pi)$ lives on a circle, not a line, and this automatically forces electric charge into integer units. In the Standard Model, quantisation is postulated. Here it is a theorem: causal graph $\to$ compact $\mathrm{U}(1)$ $\to$ quantised charge.

14.5 The causal cell: $d = 4 \to K_5$

In d-dimensional spacetime the smallest causal cell is a d-simplex with $d + 1$ vertices. Causality requires that every pair be connected $\to$ complete graph K_{d+1}. The gauge field lives on edges (connection); the field strength lives on triangular faces (curvature).

Closure principle. In the path integral $Z = \int \prod d\theta \, \exp\left(-\beta \sum (1 - \cos \Phi)\right)$, the variables are edge phases and the constraints are face holonomies. The theory is exactly defined when the number of variables equals the number of constraints: $\binom{d+1}{2} = \binom{d+1}{3}$.

d	edges $\binom{d+1}{2}$	faces $\binom{d+1}{3}$	match?
2	3	1	no
3	6	4	no
4	10	10	**yes**
5	15	20	no
6	21	35	no

Unique solution: $d = 4$, $n = 5 \to K_5$. The complete graph on 5 vertices; the 1-skeleton of the 4-simplex.

Combinatorics of K_5: 5 vertices, 10 edges, 10 faces, 5 tetrahedra. Of the 10 edges, 4 are fixed by gauge $\to 6 = \binom{4}{2}$ physical degrees of freedom. Of the 10 faces, 4 are related by Bianchi identities $\to$ 6 independent components of $F_{\mu\nu}$.

Theorem 14.3 (Hodge closure)**.** *The following four properties are equivalent and hold iff $d = 4$:*

(P1) *Minimal dynamical closure: F and $*F$ have the same form degree ($\deg F = \deg *F = 2$ iff $d - 2 = 2$).*

(P2) *Chiral closure: Λ^2 admits a self-dual/anti-self-dual decomposition.*

(P3) *Topological closure: $F \wedge F$ is a top form ($\deg 4 = d$ iff $d = 4$).*

(P4) *Simplicial duality: $\binom{d+1}{2} = \binom{d+1}{3}$.*

All reduce to $d - 2 = 2$.

Physical content: P1—the Bianchi sector ($dF = 0$) and the response sector ($d{*F} = J$) live in the same space of forms. P2 gives chirality (§16). P3 gives topological charge (§19.3).

Theorem 14.4 (Completeness of the minimal causal-gauge cell)**.** *Let $\mathcal{C}$ be a finite local cell satisfying:* (1) *the transporter lives only on edges;* (2) *Wilson dynamics;* (3) *every pair of vertices has a primitive datum without intermediaries;* (4) *minimality. Then the 1-skeleton of $\mathcal{C}$ is a complete graph K_n. With Hodge closure, the unique solution is K_5.*

Theorem 14.5 (Gauge consistency on K_5)**.** *If the curvature $F_{ijk} = 0$ for all 10 triangles, the configuration is pure gauge ($\theta_{ij} = t_i - t_j$). Any non-trivial configuration necessarily contains curvature.*

Verified: $\mathrm{rank}(D^\top D) = 6$, null space = gauge space ($\dim 4$).

Physical interpretation: a system of 5 events cannot carry information without curvature. Non-trivial dynamics inevitably manifests as force.

14.6 Uniqueness of K_5

K_5 is the unique complete graph simultaneously satisfying all necessary conditions:

Condition 1: $d = 4$. $\binom{n}{2} = \binom{n}{3}$ only for $n = 5$. Additionally: A_n has a pair of inequivalent 3-dimensional irreps (required for chirality) only for $n = 5$; spinor dimension $2^{d/2} = d$ only for $d = 4$.

Condition 2: Gauge group. $K_3 \to A_3 = \mathbb{Z}_3$ (too small). $K_4 \to A_4$ (SU(2), but not SU(3)). $K_5 \to A_5$ (SU(3), with breaking to SU(2) × U(1)). $K_6 \to \alpha^{-1} = 360$ (incompatible with physics).

Condition 3: Fermion spectrum. Only K_5 gives $5 + 10 =$ dimensions of $\bar{5} \oplus 10$, coinciding with one SM generation in SU(5) GUT. K_4 gives $4 + 6$; K_6 gives $6 + 15$—neither reproduces the SM.

Condition 4: Anomaly cancellation. Only with $\bar{5} \oplus 10$ (K_5) does anomaly cancellation yield a unique solution—SM charges.

Condition 5: Strong CP. Sym^2(face rep) for S_5 contains exactly 1 copy of the sign representation $\to$ a unique pseudoscalar Q, and $\bar{\theta} = 0$ is protected.

Condition	K_3	K_4	K_5	K_6	K_{7+}
$d = 4$ (edges = faces)	✗	✗	✓	✗	✗
Chirality (Out $= \mathbb{Z}_2$)	✗	✗	✓	✓	✗
Spinors ($2^{d/2} = d$)	✗	✗	✓	✗	✗
SU(3) gauge	✗	✗	✓	?	?
Fermions $\bar{5} \oplus 10$	✗	✗	✓	✗	✗
SM uniqueness	✗	✗	✓	✗	✗
$\bar{\theta} = 0$	—	—	✓	?	—

The intersection is a single point. Not a fit, but an intersection of five independent physical constraints.

14.7 Why the same A_5 code closes in space but grows in time

The abstract A_5 code appears in many closed spatial structures: icosahedral molecular shells (C_{60} fullerenes, viral capsids, clathrin cages, boron clusters). This does not mean that those systems and the K_5 causal window are the same object. The point is more precise: the same symmetry code behaves differently depending on whether it acts in reversible space or in an oriented causal graph.

In an ordinary spatial system the graph is effectively undirected. Rotations and rearrangements have inverses. The free-energy problem is solved within a group action, and symmetric closed orbits are allowed. This is why icosahedral A_5 structures naturally form closed finite shells.

The K_5 causal window is different. It lives first in a directed acyclic graph. The causal arrow forbids a future-to-past inverse move. Therefore the time evolution of the window is not a group action but an oriented continuation:

$$K_5^{(n)} \longrightarrow K_5^{(n+1)}.$$

Fixing a causal direction breaks the A_5 symmetry of the unoriented window to the tetrahedral stabiliser A_4. Spatial substructures can still close: this is what happens in k=3 baryonic closures (§26.4) and in future-sealed horizon clusters (§22.4). But closure along the causal arrow would be a time loop and is forbidden by the DAG postulate.

Thus K_5 does not forbid closure. It forbids temporal closure. The same closure principle that makes finite symmetric shells in space becomes continued growth when projected along the causal direction.

15 The Sliding Window

15.1 K_5 as a sliding window and 3+1

K_5 is not a "brick" of spacetime but the minimal sliding window of the causal stream. K_5 relates to the stream of events as a chord relates to a melody: not a wall element, but the minimal fragment over which the relations between successive moments first become physically meaningful.

Given a sequence of events (ticks) $e_0 < e_1 < e_2 < \cdots$, the cell $K_5^{(n)} = \{e_n, \ldots, e_{n+4}\}$ is the minimal window on which the phase structure of comparisons closes. When time advances by one tick, the window slides and the new K_5 shares a tetrahedral face (K_4) with the previous one. Temporal gluing is a consequence of sliding, not a separate postulate. Time is primary as the continuation of the stream. Space emerges secondarily as the synchronisation of multiple streams through their shared windows.

3+1 from the window. In the window $\{e_0, \dots, e_4\}$: the face without the earliest tick (tet_0) is the future; the face without the latest (tet_4) is the past. The three remaining faces contain both the earliest and latest ticks, skipping one intermediate tick each—these are the spatial channels. Each channel = one way to "skip a tick" = one of three spatial directions. 1 temporal + 3 spatial = 3+1 from the combinatorics of the 5-tick window.

Signature. The DAG singles out one direction—that of the arrows. Fixing v_0 (earliest) $\to$ stabiliser $= A_4$. $A_4 = V_4 \rtimes \mathbb{Z}_3$: V_4 is the discrete half-turn kernel of the spatial-channel algebra; spinors arise from its binary cover $Q_8 \subset 2T$ (§17.4). The $\mathbb{Z}_3$ factor cycles the three spatial channels and underlies generation triality. The full SU(3) colour structure arises at the cell-algebra level, not from $\mathbb{Z}_3$ alone (§18). Three generations are not an abstract subgroup but three ways to skip an intermediate tick in the 5-event window.

Formal results of the sliding window.

Theorem 15.1. *$K_5^{(n)} \cap K_5^{(n+1)}$ = shared K_4. In $K_5^{(n)}$ this is tet_0 (future); in $K_5^{(n+1)}$ it is tet_4 (past). The vertex map is identical to the temporal gluing rule.*

$\mathbb{Z}_3$ from the window: causal fix (v_0 = earliest) $\to A_4$. The subgroup fixing v_4 (latest) $= A_3 = \mathbb{Z}_3$ = symmetry of the three spatial channels.
Temporal face saturation: if the causal stream has no terminal tick, tet_0 of each window is closed by the next. A consequence of causal extendibility.

16 Direction Space and $\alpha = 1/137$

16.1 Direction space and $\alpha = 1/60$

The gauge field is a connection: $\theta_{ij} = -\theta_{ji}$. A "direction" is an oriented local frame: a way to parametrise the field on K_5. The automorphism group of K_5 is S_5, of order 120. Odd permutations reverse the orientation (charge conjugation). The even ones remain: A_5, of order 60.

Theorem 16.1. *$D \cong A_5$, $|D| = 60$.*

Proof. By linearity and holonomy preservation, C permutes edges; by Whitney's theorem [2] this is induced by a vertex permutation; $\det > 0$ selects the even ones: A_5. Verification: 60 matrices C_g (6×6), $\det = +1$, all distinct, $C_g C_h = C_{gh}$, $C_g^\top M C_g = M$. □

Discrete Gauss law. In the continuum, $\alpha^{-1} = 4\pi$ (the volume of S^2). In the discrete theory, the role of S^2 is played by $D \cong A_5$—the space of distinguishable orientations of the sliding window on the causal stream. The unique invariant measure on a finite group is the uniform (Haar) measure: $\alpha = 1/|D| = 1/60$. With $e = 1$: $\alpha^{-1} = 60$. This is not an analogy but the same Gauss law with a discrete direction space replacing the continuous one.

Lattice coupling: $\alpha = 1/(4\pi\beta) \to \beta = 60/(4\pi) = 4.775$—deep in the Coulomb phase.

Representation-theoretic reinforcement. By Burnside's theorem, $60 = 1^2 + 3^2 + 3'^2 + 4^2 + 5^2$—the sum of squared dimensions of all irreps of A_5. Each irrep contributes d_ρ^2 channels: 1 (trivial/phase) + 9 (chirality L) + 9 (chirality R) + 16 (Higgs/breaking) + 25 (metric). The coupling $\alpha = 1/60$ means: one quantum of curvature is distributed over 60 independent sector channels. Why A_5 and not $S_5 = 120$: only even permutations preserve orientation (chirality).

16.2 UV scale from the K_5 spectrum

The normalised Laplacian of K_5 has spectrum $\{0\,(\times 1),\ \frac{5}{4}\,(\times 4)\}$. The spectral gap $\lambda_1 = (d+1)/d = 5/4$ determines the scale at which the discrete structure is resolved:

$$\Lambda_{\mathrm{UV}} = \frac{M_{\mathrm{Pl}}}{\sqrt{\lambda_1}} = M_{\mathrm{Pl}}\sqrt{\tfrac{4}{5}} = 0.8944\, M_{\mathrm{Pl}} = 1.092 \times 10^{19}\ \mathrm{GeV}. \tag{10}$$

16.3 Running: $60 \to 137$

Quantum fluctuations screen charge. Standard one-loop QED:

$$\alpha^{-1}(m_e) = 60 + \frac{2}{3\pi}\sum_f N_c\, Q_f^2 \,\ln\frac{\Lambda_{\rm UV}}{m_f} = 60 + 77.04 = 137.041. \tag{11}$$

The nine SM fermions $(e, \mu, \tau, u, d, s, c, b, t)$ give a total contribution of +77.04. Result: 137.041 vs 137.036 (CODATA), $\Delta = 0.004\% = 37$ ppm.

Scheme invariance: four renormalisation schemes give $\alpha^{-1}_{\rm bare} = 59.8 \pm 0.1$. The integer 60 is the unique value compatible with experiment ($59 \to 136.04$, $61 \to 138.04$). Honest precision including two-loop (+0.13) and threshold matching (± 0.03): 137.0 ± 0.2 (0.1%).

16.4 The arrow of time

The arrow of time is not an emergent thermodynamic accident in the K_5 construction. It is already present in the first postulate: causal order is a directed acyclic graph. A future event cannot be used as a cause of its own past. Therefore the elementary update

$$K_5^{(n)} \longrightarrow K_5^{(n+1)}$$

has a direction.

This should be distinguished from the thermodynamic arrow. Entropy growth, coarse-graining, and irreversibility are later collective phenomena. The causal arrow is more primitive: it is the direction in which the window can continue without forming a causal loop.

In short, K_5 does not explain the arrow of time by appealing to entropy. It uses the causal arrow as part of the definition of physical continuation. Thermodynamic irreversibility is then a property of ensembles of such continuations, not the origin of time itself.

17 Chirality and Spinors

17.1 $\Lambda^2 = 3 \oplus 3'$: chirality

The 6 physical degrees of freedom = $\Lambda^2(\mathbb{R}^4)$, irrep of A_5. Character: $\chi_{\Lambda^2} = [6, -2, 0, 1, 1]$. $\Lambda^2 = 3 \oplus 3'$. $SO(4) \cong SU(2)_L \times SU(2)_R$: $3 = \mathrm{adj}(SU(2)_L)$, $3' = \mathrm{adj}(SU(2)_R)$.

A_5 is the smallest finite group with a pair of non-isomorphic conjugate 3D irreps. Chirality is a structural property of the 4D simplex.

What distinguishes 3 from $3'$? Two classes of 5-cycles in A_5 (12 each). A 5-cycle = complete traversal = causal chain. Characters: $\chi_3 = \varphi/\bar{\varphi}$, $\chi_{3'} = \bar{\varphi}/\varphi$ (φ = the golden ratio). The arrow of time makes 3 "causally aligned", $3'$ not. The sole distinction is the character on causal chains.

Gauge fixing erases the distinction: $A_5 \to A_4$, $\chi_3|_{A_4} = \chi_{3'}|_{A_4}$. All gauge-fixed objects are $L \leftrightarrow R$ symmetric. Covariant projectors P_3, $P_{3'}$ via group averaging: $P^2 = P$, $PP' = 0$, $P + P' = I$.

Chirality only in $d = 4$: A_n has $3 \neq 3'$ only at $n = 5$. For $n \leq 4$: A_n is too small. For $n \geq 6$: 3 and $3'$ are not singled out. Self-consistency: $\binom{5}{2} \times \binom{4}{2} = 60 = |A_5|$. The condition edges $\times$ dof $= |A_{d+1}| \Rightarrow d/2 = (d-2)!$ holds only for $d = 2$ and $d = 4$.

Chirality without SSB: $\chi_{\cos} \equiv 0$ identically (any gauge); $\chi_{\sin} \equiv 0$ for SU(2); $J_{\rm eff}/J_c \approx 0.76$ (subcritical). Chirality is a kinematic fact, not a dynamic one.

17.2 Spinors and Theorem T7

$2A_5 = SL(2, 5)$, $|2A_5| = 120$, $2A_5 \subset SU(2)$. Spinorial 2D irreps $\to$ fermions. The chain: $K_5 \to A_5 \to 2A_5 \subset SU(2) \to$ spinorial reps $2, 2' \to$ fermions. Every step is forced.

Theorem 17.1 (T7)**.** $\mathrm{Sym}^2(2) = 3$, $\mathrm{Sym}^2(2') = 3'$.

Proof. (1) Sym^2 homomorphism ✓; (2) irreducibility (Schur) ✓; (3) trace values $= \chi_3$ not $\chi_{3'}$ ✓; (4) $\mathrm{Sym}^2(2') = 3'$ by conjugation of A_5 irreps. Verification: mult = 1.000000. □

Physical meaning: spinor $2 \to$ gauge sector 3 ($\mathrm{SU}(2)_L$); spinor $2' \to 3'$ ($\mathrm{SU}(2)_R$). Mixing is forbidden by the algebra.

Yukawa: $2 \otimes 2 = 1 + 3$. $\langle 2 \otimes 2, 3 \rangle = 1$, $\langle 2 \otimes 2, 3' \rangle = 0$.

17.3 Tensor products and the Higgs algebra

$2 \otimes 4 = 2' + 6$—the Higgs switches spinor chirality. $2 \otimes 3' = 6$—cross L–$R \to$ only the heavy 6-irrep. $3 \otimes 3' = 4 + 5$—no direct cross-coupling. $\Lambda^2(3) = 3$, $\Lambda^2(3') = 3'$—each SU(2) is self-contained.

The K_5 Higgs: (1) VEV in $3' \to W_R$ massive. $\mathrm{mult}(1 \in 3'^3) = 1$, $\mathrm{mult}(1 \in 3' \cdot 3 \cdot 3') = 0$. (2) VEV in the bidoublet $4 \to W_L$ massive, Yukawa masses. $4|_{A_4} = 1 + 3$. The 6-irrep: all cross-L/R modes go into the heavy 6. If 6 has a mass gap $\sim \Lambda_R$, the IR theory is purely chiral.

Higgs structure: exactly one scalar. The scalar field lives in the 4-irrep of A_5. Under causal breaking $A_5 \to A_4$: $4|_{A_4} = 1 \oplus 3$. 3 = Goldstones (W^+, W^-, Z); 1 = radial mode = the physical Higgs boson. No other irrep gives this pattern: 3-irrep contains no singlet; 5-irrep gives two singlets (excluded).

The more precise leading-order result is $m_H/v = 3/\sqrt{35}$ via closure inverse stiffness (§26.5).

After the discovery of causal masses (§19.11), the Yukawa couplings $y_f = m_f/v$ are derived quantities, not fundamental parameters. The Higgs is necessary for EW breaking (W/Z masses), but its role as the origin of fermion masses is replaced by the causal mechanism.

17.4 What is spin in K_5

In K_5, spin is not the rotation of a small object in space. Space itself is not primitive. Spin is the representation-theoretic memory of how a causal motif transforms under the discrete permutations of the spatial channels inside the window.

Fixing one causally distinguished event in K_5 breaks A_5 to the tetrahedral stabiliser A_4. This group decomposes as

$$A_4 \simeq V_4 \rtimes C_3. \tag{12}$$

The C_3 factor cycles the three spatial channels; it underlies the triality and generation structure. The normal subgroup V_4 (Klein four-group) consists of the three pairwise half-turn exchanges of these channels: (12)(34), (13)(24), (14)(23).

By itself V_4 is not yet spin. Spin appears when this discrete spatial algebra is lifted to its binary cover:

$$A_4 \longrightarrow 2T \subset \mathrm{SU}(2), \tag{13}$$

where $2T$ is the binary tetrahedral group (order 24) and the preimage of V_4 is the quaternion group Q_8. In this cover a 2π rotation is represented by the central element -1, while a 4π rotation returns to $+1$. This is the K_5 origin of spinorial sign reversal.

Spin-1/2. A fermion is a motif whose external carrier transforms in a two-dimensional projective representation of the A_4 spatial-channel algebra—equivalently, in an ordinary two-dimensional representation of its binary cover $2T$. The sign change under 2π is not a postulate; it is a property of the covering map.

Spin-1. The photon is a coherent U(1) phase-transport mode whose long-distance residue transforms as a vector (adjoint/helicity-one carrier under $V_4 \to \mathrm{SO}(3)$).

Spin-0. The Higgs is a scalar radial mode of causal ordering—it transforms trivially under V_4.

Spin-2. The graviton-like response is a collective symmetric-tensor mode of the synchronised network, transforming in the rank-2 symmetric representation at long distance.

These spin labels refer to the long-distance projection of the motif onto the emergent rotational sector of the synchronised network. On a single K_5 cell the symmetry is discrete ($A_4/2T$); the familiar SO(3)/SU(2) language is the IR collective limit.

Thus spin is not added to K_5. It is the projective representation theory of the spatial-channel permutations already present in the causally oriented window.

18 Colour, SU(3), and Embedding

18.1 Colour from causality

Theorem 18.1 (Colour from causality)**.** *The arrow of time selects $A_5 \to A_4$. The tetrahedra decompose as $5 = 1 + 1 + 3$. The triplet $\{T_2, T_3, T_4\}$ = three colours. $\mathbb{Z}_3 = \langle(234)\rangle$ permutes them cyclically.*

$A_4 = V_4 \rtimes \mathbb{Z}_3$: V_4 is the spatial half-turn kernel (spin via binary cover $Q_8 \subset 2T$, §17.4); $\mathbb{Z}_3$ cycles spatial channels (generation triality; full SU(3) from cell algebra). S_3 = Weyl(SU(3)) = edge stabiliser in A_5. $3|_{\mathbb{Z}_5} = 1 \oplus \omega \oplus \omega^4$—three distinct $\mathbb{Z}_5$ charges. $\Lambda^2(3_{A_4}) = 3_{A_4}$—self-duality. $3 \otimes 3 - 1 = 8$ = adjoint. The 6 edges of the tetrahedron $= 2 \times 3_{A_4}$. SU(3) is the unique group with Weyl $= S_3$, rank = 2, fund = 3.

The full SM gauge group from K_5:

SM	K_5 object
U(1)	phase on DAG edge
$\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$	$\Lambda^2(4_{A_5}) = 3 \oplus 3'$
SU(3)	3_{A_4} on tetrahedra

Fixing the arrow of time ($A_5 \to A_4$) simultaneously breaks $L \leftrightarrow R$ and singles out 3 colours.

18.2 Induced SU(3): cell algebra

Theorem 18.2 (Cell algebra)**.** *K_5 with 10 U(1) edge phases. Gauge reduction: $10 - 4 = 6$ physical DOF on the residual K_4. The 6 edges of K_4 partition into exactly 3 perfect matchings (K_{2n} has $(2n-1)!!$; K_4: $3!! = 3$). Each matching → one root pair $(E_\alpha, E_{-\alpha})$. Commutators $[E_\alpha, E_{-\alpha}] \to$ 2 Cartan generators. Total: $6 + 2 = 8 = \dim \mathfrak{su}(3)$. The Killing form $K_{ab} = -3\delta_{ab}$ (equal root lengths) uniquely identifies $A_2 = \mathfrak{su}(3)$. $\mathbb{Z}_3$ permutes the 3 root pairs. Not $\mathfrak{su}(2)$ (needs 1 pair, K_5 gives 3); not $\mathfrak{su}(4)$ (needs 6 pairs).*

Full proof (10 steps, all computationally verified): (1) 10 edges. (2) rank(incidence) = 4 → 6 DOF. (3) rank(coboundary) = 6, Bianchi = 4. (4) K_4: 3 matchings. (5) 3 matchings → 6 roots. (6) $[E_\alpha, E_{-\alpha}] \to$ rank 2 [coefficients $(1.00, 0)$, $(0.50, 0.87)$, $(-0.50, 0.87)$]. (7) $6 + 2 = 8$. (8) Killing $= -3\delta_{ab} \to A_2$. (9) $\mathbb{Z}_3$ permutes. (10) Uniqueness among K_3–K_7.

18.3 Induced SU(3) transport

The construction imports nothing from SU(3). The fundamental dynamics involves ONLY U(1) phases θ_{ij} on K_5 edges. SU(3) transport is a derived quantity, uniquely determined by the cell gluing action $S_{\text{glue}} = \beta \sum(1 - \cos \Delta\Phi_f)$—the same form, the same β, no free parameters:

1. The shared tetrahedron between neighbouring cells has 4 faces.
2. Φ_k = holonomy of the k-th face in the target cell.

3. 4 holonomies → Hermitian traceless H via the Gell-Mann map.
4. $\tilde{U} = \exp(iH)/\det^{1/3} \in \mathrm{SU}(3)$.
5. Three spatial directions (three channels of the sliding window) → three sets of generators.

Direction	Shared tet	Direct generators
x	$\{0,1,2,3\}$	T_3, T_8, T_1, T_4
y	$\{0,1,2,4\}$	T_2, T_5, T_6, T_3
z	$\{0,1,3,4\}$	T_7, T_4, T_1, T_8

All 8 Gell-Mann generators are covered. $d \geq 3$ is necessary for the full $\mathfrak{su}(3)$—in $d = 2$, T_7 is absent (rank 7). Three-dimensionality of space is an algebraic requirement.

Shared tetrahedron = colour triplet. Upon gluing through a tetrahedron: 6 edges − 3 gauge = 3 physical DOF = one colour triplet. First the 3 colours arise as physical channels on the shared boundary; then SU(3) emerges as the gauge group—the effective description of triplet gluing on a large lattice.

Lattice verification. The induced SU(3) passes every consistency test. The algebra spans all 8 generators of $\mathfrak{su}(3)$ with complex structure (not SO(3)). Non-abelianness is unambiguous: $\langle \|[P_1, P_2]\| \rangle = 1.11 \pm 0.05$, while the abelian control gives exactly zero. Gauge invariance holds to 10^{-15}.

The theory confines. A Cornell fit to the static potential gives string tension $\sigma = 0.0109 \pm 0.0028$ (3.8σ from zero); the Creutz ratio $\chi(2,2) = 0.039 \pm 0.003$ (12.8σ). The abelian sector, by contrast, does not confine. The glueball mass ratio $m(0^{++})/\sqrt{\sigma} = 3.56 \pm 0.09$ matches SU(3) Yang–Mills (~ 3.5), and the continuum limit is reached within 2%. Step-scaling confirms the expected post-breaking signature: $\mathrm{U}(1)_Y$ runs slower than SU(2) in all 6 independent tests, consistent with heavy boson decoupling.

18.4 A_5/S_3 embedding and $\mathbb{Z}_3$ flavour structure

Lemma 18.3. *The face-to-direction mapping FS assigns 10 triangular faces of K_5 to 3 spatial directions (4 faces per direction, with overlaps).* Stab(FS) $\subset A_5$*:* $|\mathrm{Stab}| = 6 \cong S_3$ *(permutation of 3 directions).* $|A_5/S_3| = 10$ *inequivalent spacetime embeddings.*

Canonical A_5-equivariant bijection: $A_5/S_3 \leftrightarrow E(K_5) \leftrightarrow F_2(K_5)$. Each coset uniquely determines: a backbone edge (shared by all 3 direction-tetrahedra) and a complementary face (formed by the 3 missing vertices, one per direction). Edge ∪ face = $\{0,1,2,3,4\}$, edge ∩ face = ∅. The bijections are A_5-equivariant and invariant under the choice of FS representative.

Proof. Exhaustive computation over all 60 elements of A_5 and 10 orbit representatives. Each coset has exactly one shared edge and one complementary face, covering all 10 edges and all 10 faces without repetition. □

Physical interpretation: each embedding splits K_5 into backbone edge (2 vertices, gauge DOF common to all spatial directions = "spacetime skeleton") + complementary face (3 vertices, internal/flavour sector). $5 = 2 + 3$ is a forced combinatorial identity.

In the broken phase (Higgs VEV ≠ 0): backbone edge = maximally shared gauge channel. Residual S_3 = spatial isotropy. $\mathbb{Z}_2 \subset S_3$ (swap of two directions) → parity candidate.

The number $10 = \binom{5}{2} = \binom{5}{3}$ simultaneously counts: embedding sectors (A_5/S_3), edge gauge channels, and face holonomies—not three independent structures but three views of a single object.

$\mathbb{Z}_3$ **orbit decomposition:** $10 = 1+3+3+3$. $\mathbb{Z}_3 \subset A_4$ (generator: cyclic permutation of colour vertices, fixing v_0 and v_4) partitions the 10 cosets. The partition is universal for all 4 conjugate $\mathbb{Z}_3 \subset A_4$.

Orbit	Size	Backbone edges	Description
I	3	$(0,1),(0,2),(0,3)$	observer $\to$ colours (charging channels)
II	3	$(1,2),(1,3),(2,3)$	colour triangle (gluonic channels)
III	1	$(0,4)$	observer–causal axis (singlet)
IV	3	$(1,4),(2,4),(3,4)$	colour $\to$ causal (generation/flavour channels)

Full A_4 orbits: $10 = 6 + 4$ (spatial + causal). Under $\mathbb{Z}_3$: $6 \to 3+3$, $4 \to 1+3$.

Key result: colour and generations are two orbits of the same $\mathbb{Z}_3$. Not independent structures, but two orbits of $\mathbb{Z}_3 \subset A_4 \subset A_5$ on the coset space A_5/S_3.

18.5 Uniqueness of the SM fermion spectrum

Exterior algebra on K_5: $\Lambda^0 = 1$, $\Lambda^1 = 4$, $\Lambda^2 = 6$, $\Lambda^3 = 4$, $\Lambda^4 = 1$. $\Lambda^0 \oplus \Lambda^1 = \bar{5}$ (antifundamental of SU(5)). $\Lambda^2 = 10$ (antisymmetric tensor). One generation $= \bar{5} \oplus 10 = 16$ Weyl fermions $=$ exactly the SM.

The assignment is unique: only K_5 gives dimensions $5 + 10$ matching $\bar{5} \oplus 10$. K_4 gives $4 + 6$; K_6 gives $6 + 15$—neither reproduces the SM.

Anomaly cancellation: $\mathrm{SU}(3)^2 \times \mathrm{U}(1) = 0$ ✓, $\mathrm{SU}(2)^2 \times \mathrm{U}(1) = 0$ ✓, $\mathrm{U}(1)^3 = 0$ ✓, $\mathrm{Grav}^2 \times \mathrm{U}(1) = 0$ ✓, Witten SU(2): #doublets = 4, even ✓.

18.6 Three generations and charge quantisation

$N_{\mathrm{gen}} = 3$ from $\mathbb{Z}_3 \subset A_4$. No 4th generation—$\mathbb{Z}_3$ fixes exactly three orbits.

Charge quantisation: $Q = T_3 + Y \in \mathbb{Z}/6$. $Q_u = +2/3$, $Q_d = -1/3$, $Q_\nu = 0$, $Q_e = -1$. Atomic neutrality: $\sum Q_i = 3 \times (2/3) + 3 \times (-1/3) + 0 + (-1) = 0$ exactly from anomaly cancellation. Does not require SU(5) GUT.

19 Fermions and their masses

19.1 Weinberg angle: from 3/8 to 0.232 [29, 30]

At the unification scale, the Weinberg angle is determined by the representation content of one generation. The hypercharge normalisation $k_Y = \sum Y^2 / \sum T_3^2 = 5/3$ gives $\sin^2\theta_W = 3/8$. This is a theorem—the same value as in SU(5) unification, but here derived from $\bar{5} \oplus 10$ without postulating a GUT group.

The one-loop β-coefficients follow from the same spectrum: $b_1 = +41/10$, $b_2 = -19/6$, $b_3 = -7$—matching the SM exactly. The signs tell the story: U(1) grows in the IR, SU(2) shrinks, and $\sin^2\theta$ falls from 3/8 to approximately 0.23.

One-loop running with $\alpha_1^{-1} = \alpha_2^{-1} = 60$:

M_{GUT}	$\alpha_1^{-1}(M_Z)$	$\alpha_2^{-1}(M_Z)$	$\sin^2\theta(M_Z)$
M_{Pl}	85.7	40.1	0.219
$10^{17.5}$	83.4	42.0	0.232
2×10^{16}	81.5	43.4	0.242

$\sin^2\theta_W(M_Z) = 0.232 \pm 0.012$ for $M_{\mathrm{GUT}} = 10^{16..19}$ (experiment: 0.2312). The prediction is robust: $\sin^2\theta$ is a ratio, insensitive to $\alpha_{\mathrm{GUT}}^{-1}$.

Lattice confirmation. The cell architecture of K_5 (10 phases per cell, unique gluing) gives $\sin^2\theta = 0.500 \pm 0.002$ in pure gauge at all β—exact S_5 symmetry. The dressed value (0.231) is obtained analytically via fermion-driven running.

A subtlety arises with absolute couplings. $\sin^2\theta$ works (depends on b_1/b_2), but $\alpha_{\rm em}^{-1}(M_Z) = 181$ vs 128 (42% off). Cause: $\alpha_{\rm GUT}^{-1} = 60$ is $\sim 2.4\times$ the standard SU(5) value. Resolution: K_5 does NOT predict $\alpha_{\rm em}$ through unification running. Instead: (1) $\sin^2\theta$ from running ratio $3/8 \to 0.232$; (2) $\alpha_{\rm em}^{-1}(0) = 137$ from $|A_5| = 60 \to$ Gauss (§16.1); (3) α_s from lattice dynamics. Three predictions, three independent paths. "Unification" in K_5 is structural (one graph, three couplings), not SU(5)-type.

19.2 Electroweak bosons: 3 massive + 1 massless

Two-stage symmetry breaking:

First stage ($M_{\rm GUT}$): adjoint Higgs $\varphi \sim T_8$ breaks SU(3). 4 broken generators $\to$ 4 heavy gauge bosons (X, Y-type). Remaining $T_1, T_2, T_3, T_8 \to {\rm SU}(2)_L \times {\rm U}(1)_Y$.

Second stage ($v_{\rm EW} \approx 246$ GeV): Higgs doublet $H \sim (2, +1/2)$ breaks ${\rm SU}(2)\times{\rm U}(1) \to {\rm U}(1)_{\rm em}$. $W^\pm$ from $T_1 \pm iT_2$; Z^0 from $\cos\theta\cdot T_3 - \sin\theta\cdot Y$; γ from $\sin\theta\cdot T_3 + \cos\theta\cdot Y$. Number 3 massive is forced by the structure.

$\rho = M_W^2/(M_Z^2\cos^2\theta_W) = 1$ from custodial symmetry. K_5 forces the doublet via Theorem 17.1 $\Rightarrow \rho = 1$ is a consequence. $M_W = 79.9$ GeV (experiment: 80.38, $\Delta = 0.6\%$; tree-level).

19.3 Neutrinos: basic structure

The K_5 fermion content $\bar{5} \oplus 10$ includes exactly 15 Weyl fermions per generation—and no right-handed neutrino. The representation $(1, 1, 0)$ simply does not appear in $\Lambda^0 \oplus \Lambda^1(K_5)$.

The consequence is immediate: neutrinos are Majorana particles. Without ν_R, Dirac masses are impossible, and the only available mechanism is the Weinberg dimension-5 operator [8]. This is a sharp prediction: if neutrinoless double beta decay is absent at all sensitivities, minimal K_5 is refuted.

The flavour structure follows from A_4. The three lepton doublets $L = (L_1, L_2, L_3)$ form the triplet **3** of A_4, and the A_4-invariant Weinberg operator is determined by two basis matrices M_A and M_B. The general mass matrix $m_\nu = aM_A + bM_B$ has μ–τ symmetry built in, predicting $\sin^2\theta_{23} = 1/2$ at leading order. If a single spurion simultaneously breaks this symmetry and generates $\theta_{13} \neq 0$, a sum rule follows: $\theta_{23} - \pi/4 = \kappa\,\theta_{13}\cos\delta + O(\theta_{13}^2)$.

19.4 Neutrino mass hierarchy and PMNS mixing from K_5

Neutral-triplet residue law (LO). On a chain, residues $Z_{ss} = 2/5$, $Z_{11} = 1/15$, $Z_{22} = k^2/180$ give:

$$m_1 : m_2 : m_3 \approx 0 : 1 : 6, \qquad \frac{\Delta m_{21}^2}{\Delta m_{32}^2} = \frac{1}{35} = 0.0286. \tag{14}$$

Experiment: 0.0307 (NO). Agreement 4–7%. Zero free parameters.

TBM [9] from $\mathbb{Z}_3$ + CP-even Majorana: $\nu_2 = e_s = (1,1,1)/\sqrt{3}$, $\nu_1 = \sqrt{2}\,{\rm Re}(e_1) = (2,-1,-1)/\sqrt{6}$, $\nu_3 = \sqrt{2}\,{\rm Im}(e_1) = (0,1,-1)/\sqrt{2}$. LO PMNS: $\sin^2\theta_{12} = 1/3$, $\sin^2\theta_{23} = 1/2$, $\sin^2\theta_{13} = 0$. Equivalently: $\cos^2\theta_{12}\cos^2\theta_{13} = 2/3$ (experiment: 0.681, 2.1%).

19.5 Electron mass: minimal topological defect

Why does the electron exist?

The ordered phase of the K_5 lattice—the vacuum—is featureless: all holonomies small, the network relaxed. But the spectrum of configurations includes local modifications of face weights around a single edge. Among all such modifications, one is distinguished: the smallest that is stable. It modifies exactly three faces (the star of one edge), it has a strictly positive Hessian (all eigenvalues above 5, no zero modes), and it is unique. We call it Type B.

This defect is the electron. It is a non-trivial phase pattern, carried losslessly along the causal stream by the protected mode Φ_{123} of the boundary transfer (§19.8). Its mass is the cost of maintaining it against the vacuum:

$$m_e = M_{\text{Pl}} \cdot \exp(-S_{\text{eff}}). \tag{15}$$

The effective action has three contributions, each of a different origin:

$$S_{\text{eff}}(e) = \underbrace{51}_{\text{topological}} + \underbrace{0.468}_{\text{spectral}} + \underbrace{0.060}_{\text{network}} = 51.528. \tag{16}$$

The first, $S_0 = |A_5| - |E(K_5)| + 1 = 60 - 10 + 1 = 51$, counts the directions of the symmetry group minus the links of the graph plus one—a topological invariant. The second is the log-determinant ratio of defect and vacuum Hessians on a single cell. The third, $\delta S = 3/50$, is the NLO response of the neighbouring cell: the Type B defect lives on a non-shared face, perturbs two shared faces by $\Delta G = 1/10$ each, and the resulting correction $(3/5) \times (1/5) \times (1/2) = 3/50$ is an exact rational.

The experimental value is $S_{\text{eff}}^{\text{exp}} = -\ln(m_e/M_{\text{Pl}}) = 51.5278$. The discrepancy is 0.0002—a 0.02% agreement in mass, from three K_5 numbers and nothing else.

19.6 Three generations: defects and masses

Electron, muon, and tau are three particles with the same charge but different masses. On K_5, different defect types correspond to different nodes of the Hamiltonian cycle. Complete enumeration gives three distinct node determinants: $\{1, 3, 5\}$. A fourth (det = 7) is topologically forbidden—the corresponding face configuration is not a valid Type B defect. Three determinants → exactly three generations.

Muon mass (heuristic): $m_\mu/m_e = \det(3_1) \cdot \alpha^{-1}/2 = 3 \times 137/2 = 205.5$ (experiment: 206.8, $\Delta = 0.6\%$).

The Dirac chain (§19.7–§19.12) gives an independent derivation $m_\mu/m_e = 3L^{*2}/\pi^2 = 205.5$ through causal barriers—a different mechanism producing the same number. The relation between these two derivations is an open question.

Quark masses cannot be addressed as isolated numbers: a single-quark propagator is not a physical observable (§26.4). Within-sector ratios of quarks and leptons coincide at the single-chain level; the difference is a collective phenomenon.

19.7 Simplicial Dirac operator

The simplicial Dirac operator $D = d + d^*$ acts on the full cochain space of K_5, which has dimension 31. It requires no imported structure—no staggered phases, no γ-matrices, no Lorentz group. Everything is determined by the combinatorial boundary operators.

On a single cell, the spectrum has a remarkable simplicity: $\lambda = -\sqrt{5}$ with multiplicity 15, one zero mode, and $\lambda = +\sqrt{5}$ with multiplicity 15. The unique zero mode is the constant 0-form (H^0 of the contractible 4-simplex). The chiral grading $\gamma = (-1)^p$ anticommutes with D exactly: $\{D, \gamma\} = 0$. Even forms ($\Omega^0 \oplus \Omega^2 \oplus \Omega^4$): dim 16; odd ($\Omega^1 \oplus \Omega^3$): dim 15.

On a temporal chain, the Bloch Hamiltonian is a 16×16 Hermitian matrix with 16 bands in chiral pairs. Bands 7 and 8 touch at zero energy—the theory is gapless—with linear dispersion $\varepsilon = \pm\sqrt{30} \cdot k$. The velocity $\sqrt{30} = \sqrt{1^2 + 2^2 + 3^2 + 4^2}$ encodes the four temporal distances in the window. Two zero modes at $k = 0$: $H^0 \oplus H^1$ of the circle.

Exact directional decomposition. $D = D^{(0)} + D^{(1)} + D^{(2)} + D^{(3)} + D^{(4)}$, where $D^{(j)}$ collects coboundary terms with vertex v_j. The decomposition is exact (error = 0). On the 31-dimensional space: $D^{(4)}$ commutes with all of S_3 (exact colour singlet); $D^{(1)}$ with P_{23}, $D^{(2)}$ with P_{13}, $D^{(3)}$ with P_{12}; $D^{(0)}$ commutes with all of S_3. Logic: v_j belongs to exactly those spatial tetrahedra that do not skip v_j; v_4 is in all three, giving full S_3.

T-covariance. R: $v_j \leftrightarrow v_{4-j}$, $R^2 = I$, $RDR = D$ (exactly), $RD^{(j)}R = D^{(4-j)}$. The full Dirac is T-invariant; past and future are T-conjugate.

S_3 breaking at finite k. $D(k)$ does not commute with S_3 at $k \neq 0$; at $k = 0$ it commutes exactly. The breaking arises from inter-cell Bloch phases—a physical finite-momentum anisotropy.

A selection rule emerges that will prove decisive for the mass hierarchy. In the S_3-adapted generation basis (e_s, e_1, e_2), the zero-momentum Dirac matrix satisfies $D_0(e_s, e_2) = 0$ exactly, while $D_0(e_s, e_1) = -4.08 \neq 0$. Generation 2 is exactly decoupled from the singlet at $k = 0$. At finite k: $|A(e_s, e_1)|/|A(e_s, e_2)| \approx 12$—a robust 2+1 channel structure. This is the algebraic origin of the electron's lightness.

19.8 Boundary transfer kernel

When the sliding window advances by one tick, what survives?

The shared K_4 boundary carries 6 edge phases, 3 of which are gauge. The remaining 3 are physical holonomy modes. The Hodge Laplacian on K_4 edges is $\Delta_1 = 4I$—all three physical modes have the same stiffness. Local filling is exactly S_3-symmetric: no generation is preferred within a single window.

The inter-window transfer has singular values

$$\sigma(T) = \{1,\ \tfrac{1}{4},\ \tfrac{1}{4}\}. \tag{17}$$

The protected mode ($\sigma = 1$) is the colour-face holonomy Φ_{123}—the phase accumulated around the triangle of colour vertices $\{v_1, v_2, v_3\}$. Its three edges are shared between past and future boundaries and are never integrated over. The transfer is therefore lossless: transmission equals 1.0000 at every value of β, a topological fact independent of the coupling.

The two suppressed modes ($\sigma = 1/4$) are orthogonal to Φ_{123} and live in the spatial holonomies. The ratio $\sigma_1/\sigma_2 = 4 = |K_4|$ is a topological invariant of the window structure.

The pattern $\{1, 1/4, 1/4\}$ is reproduced identically at every step of the chain. It is not a dynamical accident but an emergent law of the sliding window.

One-step filling. Adding v_4 to the past boundary $K_4 = \{v_0, v_1, v_2, v_3\}$ completes K_5. Gaussian integration over 4 new edges gives effective boundary action $M_{\text{eff}} = P_\perp$ with eigenvalues $\{0, 0, 0, 1, 1, 1\}$. Total kernel: $\{0, 0, 0, 5, 5, 5\}$—all three physical holonomies receive the same scale $\sqrt{5}$.

Finite lifetime of the colour face. $\Phi(n{+}1, n{+}2, n{+}3)$ of window n uses edges $\{(n{+}1, n{+}2), (n{+}1, n{+}3), (n{+}2,$
They survive in window $n{+}1$, but in window $n{+}2$ vertex $n{+}1$ leaves K_4, killing two of three edges. Lifetime = 2 windows. Successive colour faces share only 1 of 3 edges; after 3 steps the set is completely refreshed. $\sigma = 1$ is a one-step lossless transfer, not infinite protection.

Origin of the number 4. $B^\top B =$ graph Laplacian of K_4. Its nonzero eigenvalue is $4 = |V(K_4)|$. The ratio $\sigma_1/\sigma_2 = 4$ is a topological invariant: the boundary mass equals the number of boundary vertices.

Physical interpretation. The boundary K_4 carries 3 holonomies. Local filling does not distinguish them (eigenvalue 5). Inter-window transfer does: the all-colour Φ_{123} is lossless, the spatial holonomies are suppressed by 4. This is the boundary realisation of the 1+2 pattern found independently in the Dirac sector (§19.7).

19.9 IR structure and zero-mode residues

The Dirac selection rule $D_0(e_s, e_2) = 0$ has a gauge-sector counterpart. The infrared mode of the temporal chain, projected onto the colour basis, gives $\langle \mathrm{IR}, e_s \rangle = 6.93$, $\langle \mathrm{IR}, e_1 \rangle = \sqrt{2}$, and $\langle \mathrm{IR}, e_2 \rangle = 0$—exactly, from the identity $5 - 2 \cdot 4 + 3 = 0$. Channel e_2 has zero overlap with the massless branch. Its fluctuations converge to a finite limit as the chain grows: $\langle t_2^2 \rangle \to 0.155/\beta$ for $L \to \infty$.

Theorem 19.1 (Zero-mode residues)**.** *On the temporal K_5-chain, the Bloch propagator restricted to the generation-edge basis has residues*

$$Z_{ss} = 2/5, \qquad Z_{11} = 1/15, \qquad Z_{e_2e_2} = k^2/180. \tag{18}$$

All three are exact rationals.

The ratio $Z_{11}/Z_{ss} = 1/6$ is k-independent. The e_2 residue vanishes at $k = 0$ and grows only quadratically—not an exponential gap but a quadratic screening, with the full analytic form $G^0_{e_2e_2}(k, z) = zk^2/(180(30k^2 - z^2))$.

This is the mechanism that makes the electron light. It is not the "smallest Yukawa coupling to a heavy scalar." It is a near-complete absence of access to the massless branch—a qualitatively different explanation. The heaviest channel is the most decoupled from the massless sector.

Gap estimate. With the scaling $D \times G$: $\text{gap}(e_2) = 0.289 \times \langle t_2^2 \rangle$. At $\beta = |A_5|/(4\pi) \approx 4.77$: $\text{gap}(e_2) \approx 9.4 \times 10^{-3}$ in lattice units—a self-consistent one-step estimate.

Three manifestations of e_2 selection. The same e_2-decoupling appears in three independent calculations: (i) Dirac selection rule $D_0(e_s, e_2) = 0$ (§19.7); (ii) protected mode Φ_{123} (§19.8); (iii) IR covariance $\langle \text{IR}, e_2 \rangle = 0$ (this section). The first and third are exact zeros from the same identity $(5 - 2 \cdot 4 + 3 = 0)$. The second is consistent; the holonomy→flavour intertwiner requires further clarification.

19.10 Fermion transfer and splitting architecture

The 31-dimensional cochain space splits canonically as $\Omega = S \oplus P \oplus F \oplus I$ (shared memory, past-only, future-only, interior: $7+8+8+8$), and the transfer operator is exactly S_3-equivariant.

Restricted to the flavour triplet—past generation edges $(0,1), (0,2), (0,3)$ and their future counterparts $(1,4), (2,4), (3,4)$—the transfer takes a striking form:

$$T_{\text{flav}}(z) = \frac{-1}{4 - z^2} \times I_3. \tag{19}$$

This is the resolvent of the Hodge Laplacian on K_4 with a pole at $z = \pm 2$. All three generations have the same boundary mass. A single window does not split flavours.

Where does the splitting come from? Not from a single cell but from the chain. Three layers contribute, each breaking less symmetry than the one before:

At the level of one window, S_3 is exact. At $k = 0$ on the chain, the Dirac selection rule $D_0(e_s, e_2) = 0$ creates a 2+1 pattern. At $k \neq 0$, the inter-window holonomy $\sigma = \{1, 1/4, 1/4\}$ and the IR screening complete the hierarchy.

The bridge formula $Z_{11}/Z_{22}(k_{\min}) = 3L^2/\pi^2$ grows quadratically with chain length and matches the observed $m_\mu/m_e = 207$ at $L = 26$. The structural constant $Z_{ss}/Z_{11} = 6$ is independent of L.

Two mass scales from K_5. Boundary: $m_\partial = \sqrt{4} = 2$ (Hodge K_4). Bulk: $m_{\text{bulk}} = \sqrt{5}$ (Hodge K_5). Ratio: $m_\partial/m_{\text{bulk}} = 2/\sqrt{5} = \sqrt{4/5}$—a number from K_5, not a parameter. Both are S_3-exact. Splitting comes not from one window but from the temporal chain.

Free pole basis. Diagonalisation of the 3×3 matrix $G^{\text{rad}}(z)$ singles out three eigen-operators with distinct first poles:

Eigen-operator	First pole λ^2	Composition
$u_{\sqrt{5}}$	5	$\approx -0.378\, e_s + 0.926\, e_1$
$u_{2\sqrt{2}}$	8	$= e_2$ (exactly)
u_3	9	$\approx 0.775\, e_s + 0.632\, e_1$

In channel language: $m(e_s) = m(e_1) = \sqrt{5}$, $m(e_2) = 2\sqrt{2}$. In eigen-operator language: $\sqrt{5}$, $2\sqrt{2}$, 3. No contradiction—different bases.

With dressed IR mass: $Z_{11}/Z_{22}^{\text{eff}} = 12/[(2\pi/L)^2 + m_{\text{IR}}^2/30]$, saturating at $L \gg \xi = \sqrt{30}/m_{\text{IR}}$ to $(Z_{11}/Z_{22}^{\text{eff}})_{\text{sat}} = 360/m_{\text{IR}}^2$. The bare 1D hierarchy grows as L^2 until generating a self-consistent m_{IR}, then saturates.

19.11 Charged lepton masses from K_5 causality

The Standard Model has two large mass ratios among charged leptons: $m_\tau/m_\mu \approx 17$ and $m_\mu/m_e \approx 207$. Their very different magnitudes suggest that no single mechanism produces both. K_5 confirms this: the two ratios arise on different causal scales.

19.11.1 Two causal barriers

Mechanism	Computes	Physical meaning
Local one-window curvature cost	$W_s = 13/3$, $W_1 = 1/4$, $W_2 = 17/12$	curvature volume seen by the channel within one window
Chain IR screening along the stream	$Z_{ss} = 2/5$, $Z_{11} = 1/15$, $Z_{22} = k^2/180$	how strongly the channel couples to the soft branch

19.11.2 Carrier identity

W and Z act not on two similar objects but on the same flavour carrier: the 3×3 generation-edge space $\mathbb{R}^3 = \{e_s, e_1, e_2\}$—Triplet IV of §18.4. This turns the factorisation from a heuristic into a causally natural candidate.

19.11.3 Causal factorisation

$[M_W, M_Z(L{=}26)]$ relative norm $= 0.125$; MC correlation $r = -0.16$. The action splits as $S = S^{\text{loc}} + S^{\text{IR}}$ at leading order; the cross-term is $O(1/\beta)$.

19.11.4 Leading causal mass law

The first ratio is local—set within a single K_5 window by the channel-resolved gauge curvature cost. The curvature weights $W_s = 13/3$, $W_1 = 1/4$, $W_2 = 17/12$ give

$$m_\tau/m_\mu = W_s/W_1 = \frac{13/3}{1/4} = \frac{52}{3} = 17.33 \qquad \text{(observed: 16.82, 3\%)}. \tag{20}$$

Note the operational status: 52/3 is the exact local curvature barrier (a structural leading-order component). The final physical pole ratio (observed: 16.82) requires the full chain/dressing corrections, making the 3% deviation a measure of NLO dressing rather than a failure of the exact local ratio.

The second is global—set by IR screening along the full causal depth $L^* = 26$:

$$m_\mu/m_e = 3L^{*2}/\pi^2 = 205.5 \qquad \text{(observed: 206.8, 0.6\%)}. \tag{21}$$

Ratio	K_5	Experiment	Agreement
m_τ/m_μ	$52/3 = 17.33$	16.82	97%
m_μ/m_e	$3 \times 26^2/\pi^2 = 205.5$	206.8	99.4%
m_τ/m_e	$52 \times 26^2/\pi^2 = 3562$	3477	97.6%

19.11.5 Structural constant 6—double derivation

$6 = Z_{ss}/Z_{11}$ (kinematics, residue ratio) $= \exp(0.475 \times (1/W_1 - 1/W_s))$ (dynamics, curvature). The number 6 appears twice—from chain kinematics and from one-window dynamics—two layers describing the same physics from different sides.

Matrix upgrade. A naive matrix hybrid (combining both mechanisms into a single matrix diagonalisation) fails: rank collapse mixes the two causally separated scales into one, pulling both ratios toward the same order. The diagonal law is the correct LO: two causally separated scales, one local (curvature) and one global (chain screening). Matrix corrections are NLO (∼12%).

Non-perturbative benchmarks. Wilson MC at $L = 26$, $\beta = 15/\pi$: $C_{ss}/C_{11} = 6.06 \pm 0.01$ (structural constant 6 confirmed); $C_{11}/C_{22} = 116 \pm 1$ vs SM target 207—gap ∼1.8×, traceable to five sources (RHMC dressing, LCP, NLO defect, flavour map, 4D dynamics).

19.11.6 Absolute scale and relative hierarchy separate

Type B defect sets the absolute cost of a charged lepton: $m_e = M_{\rm Pl} \times \exp(-S_{\rm eff}) \approx 0.52$ MeV. Curvature + screening set the relative distribution among τ, μ, e. These are two mechanisms with different causal origins: first create a rare defect, then determine how easily a given flavour channel passes along the stream.

Absolute masses: $m_e \approx 0.52$ MeV (98%), $m_\mu \approx 107$ MeV (99%), $m_\tau \approx 1853$ MeV (96%).

19.12 Causal depth $L^* = 26$ (theorem)

Theorem 19.2 (Causal depth)**.** $L^* = \dim \Omega^{\geq 1}(K_5) = 10 + 10 + 5 + 1 = 26$ *(exact combinatorial invariant).*

Structural decomposition: $\dim \Omega(K_5) = 31 = 26 + 4 + 1$ (relational + gauge + constant). Dynamical shadow: $L_{\rm dyn} = \beta\sqrt{30} \approx 26.15$. Resonant realisation: $k_{\rm IR} \cdot L^* = 2\pi$.

Three independent derivations converge to $L^* = 26$: (1) Combinatorial: $\dim \Omega^{\geq 1}(K_5) = 31 - 5 = 26$. (2) Dynamical: $\beta\sqrt{30} \approx 26.15$. (3) Resonant: $k_{\rm IR} \cdot L^* = 2\pi$.

$L^* = 26$ is not the size of the world but the reading depth of the world by a single causal stream.

19.13 Koide [7] and $\mathbb{Z}_3$

$Q = 2/3$ (experiment: 0.666656, deviation 0.002%). $M_K^2 = v_H \exp(-60/9) = 313.1$ MeV ($\Delta =$ 0.2%). All three masses to 0.3% from $\alpha^{-1} = 60$, $N_{\rm gen} = 3$, $+v_H$.

19.14 EW bilinear kernel (theorem)

The EW bilinear carrier is $\mathcal{B}_{\rm EW} = 3_L \otimes 3_R$ (dimension 9), where 3 = generation triplet (Triplet IV). Under A_4: $3 \otimes 3 = 1 \oplus 1' \oplus 1'' \oplus 3_a \oplus 3_b$.

Theorem 19.3. *The one-step K_5 kernel on $\mathcal{B}_{EW}$ with Haar averaging over A_5:*

$$T_{EW} = \frac{3}{5} P_1 + \frac{3}{10} P_{II} + 0 \cdot P_{rest}. \tag{22}$$

Proof. $T_{\rm EW}[(k,l),(i,j)] = (1/|A_5|) \sum_{g \in A_5} [g(i) = k] \cdot [g(j) = l] \cdot [k, l \in \text{FLAV}]$. For the trivial singlet $|1\rangle = (E_{11} + E_{22} + E_{33})/\sqrt{3}$: for fixed $(i,k) \in \text{FLAV}^2$, the number of $g \in A_5$ with $g(i) = k$ is $|A_5|/|V(K_5)| = 60/5 = 12$. Hence $T_{\rm EW}[(i,i),(k,k)] = 12/60 = 1/5$. Summing over 9 pairs and normalising by 1/3: $\langle 1|T_{\rm EW}|1\rangle = \frac{1}{3} \cdot 9 \cdot \frac{1}{5} = 3/5$. □

The leading eigenvalue:

$$\lambda_1 = \frac{N_{\text{gen}}}{|V(K_5)|} = \frac{3}{5}. \tag{23}$$

An exact K_5 rational—no free parameters, one number from combinatorics.

After $L^*/2 = 13$ steps: $y_{\text{EW}} = (3/5)^{13} \approx 1.55 \times 10^{-3}$.

Comparison: $(3/5)^{13}$ agrees with the K_5-derived y_K (from §19.11 + Koide) to 0.6%; $\exp(-60/9)$ from §19.13 agrees with the experimental y_K to 0.2%. The two formulae describe different levels: discrete kernel-level vs continuum/resummed envelope. The $(3/5)^{13}$ vs $\exp(-60/9)$ gap ($\sim$2%) reflects the 2% errors of the §19.11 mass chain (bare vs dressed relation), not a K_5 puzzle.

19.15 EW scale architecture: the leptonic bridge

In K_5, the EW scale $v = 246$ GeV has no independent dimensional anchor. It is derived entirely from the leptonic sector via the Koide bridge:

$$v = M_K \cdot \exp\left(\frac{|A_5|}{N_{\text{gen}}^2}\right) = M_K \cdot \exp\left(\frac{60}{9}\right), \tag{24}$$

where $M_K = (\sum_f \sqrt{m_f}/3)^2$ is the Koide mean computed from K_5-derived lepton masses. Numerically: $v = 251$ GeV (2% off experimental 246.22 GeV).

The full causal chain of the EW sector:

$$m_e \xrightarrow{S_{\text{eff}}=51.528} M_{\text{Pl}} \xrightarrow{3L^{*2}/\pi^2,\ 52/3} (m_\mu, m_\tau) \xrightarrow{\text{Koide}} M_K \xrightarrow{\exp(60/9)} v \xrightarrow{3/\sqrt{35}} m_H. \tag{25}$$

Key observation: W and H are not independent. In the SM, v is a free parameter (one of 19). In K_5, v is a derived ratio from the leptons. There is no separate "anchor for the EW sector". The sole dimensional anchor of the theory is whatever calibrates m_e (or equivalently M_{Pl} via §19.5).

	SM	K_5
v_{EW}	free parameter	derived via Koide bridge
m_H	$y_H \times v$ (y_H free)	$v \times 3/\sqrt{35}$ (§26.5)
M_W	$g_2 \times v/2$	g_2 derived from α_{K_5} and $\sin^2\theta_W$
Yukawas y_f	9 free	derived via causal chain
Hierarchy problem	unresolved	derived from leptons

Status: all EW scales (v, M_W, M_Z, m_H) derive from one anchor (m_e via §19.5 $\to M_{\text{Pl}}$). The hierarchy problem is formally resolved through the leptonic chain.

19.16 K_4 matching structure and Yukawa

After causal fixing of vertex v_0, the residual graph K_4 (vertices $\{1,2,3,4\}$) has exactly 3 perfect matchings (K_{2n} has $(2n-1)!!$; for $n = 2$: $3!! = 3$): $M_1 = \{(1,2),(3,4)\}$, $M_2 = \{(1,3),(2,4)\}$, $M_3 = \{(1,4),(2,3)\}$. The 6 edges of K_4 partition into 3 matchings of 2 edges each; the partition is unique up to relabelling.

S_4 on vertices permutes the matchings; the stabiliser of all three is $V_4 \cong \mathbb{Z}_2 \times \mathbb{Z}_2$. The induced quotient $S_4/V_4 \cong S_3$; for even permutations $A_4/V_4 \cong \mathbb{Z}_3$—cyclic permutation $M_1 \to M_2 \to M_3$. Each matching carries $\mathbb{Z}_3$ charge ω^k.

Tree-level Yukawa in the vertex basis: $Y = \sum T(b,a) \cdot T(a,b) = (3/2) \cdot I$—three generations exactly degenerate. At finite gauge field: $|Y_1| : |Y_2| : |Y_3| = 2.8 : 1 : 2.8$ at $\beta = 5$. Generation 2 is the weakest at all β, consistent with the Dirac selection rule $D_0(e_s, e_2) = 0$ (§19.7).

19.17 CKM hierarchy from A_4 [32]

Three generations = triplet **3** of A_4. With exact A_4: Y_u and Y_d are simultaneously diagonalisable $\to V_{\rm CKM} = I$ (no mixing). CKM mixing requires A_4 breaking.

The key mechanism: the up-sector preserves $\mathbb{Z}_3 \subset A_4$, the down-sector preserves $\mathbb{Z}_2 \subset A_4$. The misalignment of residual symmetries generates CKM.

With perturbative breaking parameter $\varepsilon \ll 1$: $V_{us} \sim \varepsilon$, $V_{cb} \sim \varepsilon^2$, $V_{ub} \sim \varepsilon^3$—the Wolfenstein hierarchy $\lambda : \lambda^2 : \lambda^3$.

Power-counting check: $\log|V_{cb}|/\log|V_{us}| = 2.14$ (predicted: 2, 7% off). Cabibbo hint: $\lambda_{\rm Cabibbo} = \sin(2/9) = 0.220$ (experiment: 0.225, -2.2%). A single $\mathbb{Z}_3$ phase appears to control both lepton masses and CKM.

Structural predictions: power-law $\varepsilon : \varepsilon^2 : \varepsilon^3$; 3 mixing angles from $N_{\rm gen} = 3$; one CP phase from $(3-1)(3-2)/2 = 1$; approximate μ–τ symmetry from $\mathbb{Z}_2 \subset A_4$. Quantitative: exact $\theta_C \approx 13^\circ$ and $\delta_{CP} \approx 68^\circ$ require the flavon sector (not fixed by minimal K_5).

19.18 Neutrino sum rule

$\cos^2\theta_{12}\cos^2\theta_{13} = 2/3$ (A_4 TBM first column). Origin: $A_4 \to \mathbb{Z}_3$ breaking preserves tribimaximal first column $|U_{e1}|^2 = 2/3$.

Experiment (NuFIT 5.2 NO): 0.681 ± 0.012 vs predicted 0.667—deviation 2.1%, 1.2σ. Equivalently: $\sin^2\theta_{12} = 0.318$ predicted from measured θ_{13} (experiment: 0.304 ± 0.012, 1.2σ).

JUNO test ($\sim$2027): $\sigma(\sin^2\theta_{12}) \to 0.003$ ($4\times$ improvement). If the current central value holds $\to 4.8\sigma$ tension (falsification). If $\sin^2\theta_{12}$ shifts upward to $\sim 0.318 \to$ confirmation. This is the sharpest near-term experimental test of K_5.

19.19 FCNC and charge quantisation

No tree-level FCNC: one Higgs doublet (T7) + three generations $\to$ GIM mechanism. The Z-boson coupling through T_3 and Y is diagonal in flavour space; CKM appears only in charged currents ($W^\pm$). FCNC appear only in loops, suppressed as $(m^2_{\rm heavy}/M_W^2) \times (\alpha/\pi)$.

Charge quantisation: in the standard formulation of the SM (prior to imposing anomaly cancellation), hypercharges are free parameters. In K_5: $Y \in \mathbb{Z}/6$ is an immediate structural consequence of $|\mathrm{Weyl(SU(3))}| = 6$. $Q = T_3 + Y \in \mathbb{Z}/6$. Atomic neutrality $\sum Q_i = 0$ follows from anomaly cancellation, not from SU(5) GUT.

19.20 Anomalous magnetic moment

Loop corrections in K_5 are not imported from QED—they arise as connected cumulants of the compact Wilson action. The full derivation of loop structure, the Schwinger term $a_e = \alpha/(2\pi)$, and the K_5-native origin of fermion loops is given in §24.7.

19.21 Antimatter and CPT

In K_5, antimatter is not introduced as a second species of object. It is the orientation-reversed form of the same causal motif. A charged Type B defect carries an oriented U(1) edge transporter. Reversing the orientation of the defect sends

$$g_e = e^{i\theta_e} \longmapsto g_e^{-1} = e^{-i\theta_e},$$

and therefore reverses the charge. The positron is the orientation-reversed Type B defect corresponding to the electron.

The same logic applies to the discrete spacetime operations. Charge conjugation C reverses the U(1)-orientation of the defect. Parity P acts by exchanging the spatial channels of the K_5

window. Time reversal T is not an allowed dynamical evolution of the DAG, but it is a well-defined orientation reversal of the boundary kernel. The combined operation CPT returns the unoriented phase-closure data of a stable motif to itself.

Thus K_5 does not require antimatter to be a separate ontology. Matter and antimatter are opposite orientations of the same causal motif.

20 Prohibitions

20.1 $\bar{\theta} = 0$ from the orientational symmetry of K_5

The problem. The SM allows the CP-violating term $\bar{\theta} \cdot (g^2/32\pi^2)\,\mathrm{Tr}(F\tilde{F})$. Experiment requires $|\bar{\theta}| < 10^{-10}$, but the SM does not explain this smallness—one of the oldest open problems.

Resolution from K_5. The topological charge Q is a quadratic pseudoscalar transforming in the sign representation of S_5. The gauge action and measure are invariant under the full S_5, including odd permutations. Therefore $\langle Q \rangle = 0$ exactly.

Explicit construction.

1. Face holonomies $\{\Phi_f\}_{f=1}^{10}$ form the 10-dimensional representation V of S_5.
2. $\mathrm{mult}(\mathrm{sign}, V) = 0$—no linear pseudoscalar exists.
3. $\mathrm{mult}(\mathrm{sign}, \mathrm{Sym}^2 V) = 1$—a unique quadratic pseudoscalar:

$$Q = \sum_{f_1, f_2} C_{f_1 f_2}\, \Phi_{f_1} \Phi_{f_2}, \tag{26}$$

   where $C_{f_1 f_2} \neq 0$ only for face pairs sharing exactly one vertex and covering all 5 vertices of K_5. The sign of C is determined by the Levi-Civita orientation of the 4-simplex—the lattice analogue of $\varepsilon_{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma}$.
4. Verified: $Q(\sigma \cdot \theta) = \mathrm{sign}(\sigma) \cdot Q(\theta)$ for all 120 elements of S_5 on multiple random configurations (ratio $= \pm 1.0000$ without exception).
5. **Symmetry argument.** $S = \beta \sum (1 - \cos\Phi)$ is S_5-invariant (cos is even). The measure $d\mu = e^{-S} \prod d\theta$ is S_5-invariant. For any odd τ: $\langle Q \rangle = \langle Q(\tau \cdot \theta) \rangle = \mathrm{sign}(\tau) \langle Q \rangle = -\langle Q \rangle \implies \langle Q \rangle = 0$ exactly.
6. Any term in S_{eff} proportional to Q is forbidden by S_5: its coefficient $= 0$. Therefore $\bar{\theta} = 0$.

How this differs from standard approaches. S_5 is a discrete symmetry (not continuous Peccei–Quinn U(1)), so it is not broken by gravitational effects, is not subject to quantum anomalies, and is an exact symmetry of the lattice measure at all scales. No axion field is required.

Lemma 20.1 (Stability)**.** *Let μ be any S_5-invariant measure, Q in the sign representation. Then $\int Q\, d\mu = 0$.*

Corollary 20.2. *If $S_{\mathit{eff}} = S_0 + \sum c_n O_n$ where every O_n is an S_5-invariant counterterm of arbitrary dimension, then $e^{-S_{\mathit{eff}}} |d\theta||d\varphi|$ is S_5-invariant $\Rightarrow \langle Q \rangle = 0$ for any $\{c_n\}$. The protection is absolute. The only escape is an operator breaking S_5, but $S_5 = \mathrm{Aut}(K_5)$ is exact.*

Falsifiable prediction. Neutron EDM from $\theta_{\text{QCD}} = 0$. Observable EDM is from the CKM phase only ($\sim 10^{-31}\, e \cdot \mathrm{cm}$). Detection of a $\bar{\theta}$-induced EDM at levels incompatible with CKM would falsify the mechanism.

Gap closure. S_5 acts on all sectors. On gauge: $S = \beta \sum (1 - \cos\Phi)$ depends on cos (even), hence invariant. On Higgs: adjoint action depends on traces, hence invariant. On fermions:

$D \to PDP^{-1}$, hence $\det(D)$ invariant. On the measure: $|\det(\text{permutation})| = 1$, hence invariant. There is no discrete anomaly: S_5 acts on colour indices by conjugation (vector-like, not chiral), and vector-like discrete symmetries have no 't Hooft anomalies. In the continuum: $\Phi_f \sim a^2 F_{\mu\nu}$, so $Q \sim \varepsilon_{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma} \cdot a^4 = a^4\,\mathrm{Tr}(F\tilde{F})$—quadratic in F, Levi-Civita contracted, pseudoscalar, covering all 4 directions.

20.2 Proton stability

Every known channel of proton decay is blocked. The gauge channel requires X, Y bosons in $(3, 2, -5/6)$—but K_5 induces SU(3), not SU(5), and these bosons do not exist. The scalar channel requires a coloured Higgs—but the K_5 Higgs is an adjoint EW singlet. The dimension-5 channel requires a colour triplet—none is available. The first allowed operators appear at dimension 6, giving $\tau_p > 10^{36}$ yr for $\Lambda = 10^{16}$ GeV.

Observation of $p \to \bar{\nu} K^+$ at any lifetime would directly falsify the theory.

20.3 No dark-matter particles

K_5 produces exactly the Standard Model: $(\bar{5} \oplus 10) \times 3$. No additional representations, no hidden sector, no BSM fields. Discovery of a dark-matter particle in direct detection would require extending the minimal framework.

21 Non-perturbative sector dynamics

21.1 Gauge-fixed propagator

In star gauge ($\theta_{0i} = 0$), 4 edges are fixed and 6 physical edges remain. The action $S \approx (\beta/2)\theta^\top M \theta$ with M having eigenvalues $\{1\,(\times 3),\ 5\,(\times 3)\}$. The two eigenvalues correspond to two irreps of A_5: 3 (chirality L, eigenvalue 1) and $3'$ (chirality R, eigenvalue 5). Consequence: $\langle \Phi^2 \rangle = 3/(5\beta)$—identical on all 10 faces by A_5 symmetry. The face propagator $P = V^\top M^{-1} V$ is a rank-6 projector: it projects onto the 6-dimensional physical space from the 10-dimensional face space.

21.2 One coupling, two sectors

One lattice coupling: $\alpha = 1/60$, $\beta = 4.775$—the coupling on the K_5 cell, not SU(5)-type unification. The EW and colour sectors acquire different effective couplings through breaking and running. By Schur's lemma, $M|_3 = M|_{3'}$. But by T7 (§17.2): spinor 2 sees only sector 3 via P_3, while $2'$ sees only $3'$. With $N_L \neq N_R$, emergent $g^2_{\text{eff},3} \neq g^2_{\text{eff},3'}$.

21.3 Why the effect is non-perturbative

$\det(D) \in \mathbb{R}$ ($D^\top = \overline{D} \Rightarrow$ determinant is real, $\Delta = 8 \times 10^{-16}$ on 5000 configurations).

Hessian T7: F_L eigenvalues $= \{-0.723\,(\times 3),\ 0\,(\times 3)\}$. Non-zero in sector 3, zero in $3'$. $F_L < 0$: one-loop predicts softening—the wrong sign. MC shows stiffening. The mechanism is fundamentally non-perturbative.

Logarithmic barrier: at $|\theta| = 0 \to -0.990$; $|\theta| = 0.5 \to -0.745$; $|\theta| = 1.0 \to +0.490$ ($\det \to 0$). The "dome" confines θ_3. $V_{\text{ferm}} \equiv 0$ along sector $3'$. T7 is exact.

21.4 MC: sector splitting

Config	Φ^2 ratio	Plaq ratio	Var ratio	σ
Pure gauge	1.00 ± 0.01	1.00 ± 0.01	1.00	1.8
$1L + 0R$	0.86	0.83	0.72	22
$3L + 0R$	0.69	—	0.44	62
$3L + 1R$ (SM)	0.80	0.81	0.65	30
$0L + 1R$ (mirror)	1.17	1.18	1.45	18

Pure gauge null ✓, monotonicity ✓, $L \leftrightarrow R$ mirror ✓, decoupling ✓, stability L=1..6 ✓.

The physical picture: fermions stiffen their own sector (Φ^2 decreases) and soften the opposite sector (Φ^2 increases). The SM configuration ($3L$+$1R$) has a 35% variance asymmetry between sectors—the same K_5 cell, the same β, but one sector is tighter than the other. This is the non-perturbative origin of the effective coupling split between SU(2) and U(1).

21.5 Universal equation of state

$\ln(\mathrm{Var}(O_3)/\mathrm{Var}(O_{3'}))_{\mathrm{corr}} = -(D/\beta) \cdot \Delta N$, with $D = 1.47 \pm 0.11$ (7%). $c_3 \cdot \beta = \text{const}$ across 6 values of β. The non-perturbative constant $D \cdot \beta = 1.47$.

21.6 IR screening

On an 8×8 lattice of K_5 cells with inter-cell coupling $J = 4.0$, 10 shared faces per bond, $N_m = 1500$ measurements, 300 bootstrap resamples: baseline-corrected sector ratios: pure gauge = 1.16, $3L$+$0R$ = 0.55 (53% suppression), SM = 0.70 (30% suppression).

The multi-cell lattice confirms the single-cell pattern: sector splitting persists and grows with the number of cells. The IR sector ratio approaches a fixed point from above, consistent with the universal equation of state.

21.7 Infrared Lorentz invariance

A discrete causal window appears, at first sight, to threaten Lorentz invariance. The K_5 construction avoids this in a specific way. Lorentz symmetry is not imposed as a microscopic postulate. It is the infrared symmetry of the surviving gapless transport sector of the ordered causal network.

The mechanism has three ingredients.

First, the causal arrow is primitive, but the Wilson action on the local unoriented K_5 window does not hard-code a preferred spatial axis. After a past face is selected, the remaining non-past faces form the tetrahedral A_4-symmetric local structure: one continuation channel and three spatial channels. The ordered background selects a continuation branch, while the local fluctuation operator is still governed by the symmetric gluing data of the window.

Second, the transfer operator across a shared K_4 seam separates the long-distance transport sector from heavy subleading sectors. The dominant phase-transport mode is transmitted without a mass gap ($\sigma = 1$), whereas the subleading modes acquire finite gaps of order the microscopic causal scale ($\sigma = 1/4$). In transfer language, if $\lambda_a(k) = e^{-\varepsilon_a(k)}$, then only the branch with $\varepsilon_a(0) = 0$ survives at macroscopic distance. Modes with $\varepsilon_a(0) > 0$ are exponentially suppressed and do not define the observed low-energy light cone.

Third, the K_5 window is geometrically frustrated (dihedral angle $\arccos(1/4) \approx 75.5°$, non-integer packing) and does not generate a regular crystal of preferred axes. The mismatch of simplex angles forces successive gluing frames to sample orientations rather than accumulate a fixed lattice anisotropy. This suppresses the usual preferred-axis artefacts of regular lattices. In the infrared, the surviving massless phase-transport sector therefore sees an effectively rotational continuum with universal causal speed $c = 1$ in lattice units.

Thus Lorentz invariance in K_5 is not exact microscopic lattice symmetry. It is the emergent kinematics of the unique gapless transport sector after gapped anisotropic modes and microscopic preferred-frame data have decoupled. A next-to-leading-order analysis of the transfer operator on the frustrated network is required to compute the precise Lorentz-invariance-violation signature; this is a future precision target for high-energy photon, neutrino, and cosmic-ray observations [35].

22 Gravity

22.1 Gravity as network elasticity

A massive particle is a graph defect. Around the defect the graph is "compressed" $\to$ geodesic motion = gravity. The chain: Ollivier–Ricci curvature [18] on the graph $\to$ Regge action [5] on the triangulation $\to$ Einstein–Hilbert action (unique by Lovelock's theorem [6] in $d = 4$) $\to$ Einstein equations $G_{\mu\nu} = 8\pi G\, T_{\mu\nu}$.

Theorem 22.1 (K_5 curvature)**.** *A regular lattice of identical equilateral 4-simplices cannot be exactly flat. The dihedral angle of the regular 4-simplex is* $\alpha = \arccos(1/4) \approx 75.52°$. *For zero deficit around a hinge one needs* $2\pi/\alpha \approx 4.77$ *simplices—not an integer. Curvature is not an added ingredient but a geometric consequence of regular* K_5 *gluing.*

Curvature-sign lemma: at $n_h = 4$, deficit $\varepsilon = +1.011$ rad (positive); at $n_h = 5$, $\varepsilon = -0.307$ rad (negative). Flatness is the effective average over an irregular ensemble.

Regge action: $S_{\text{Regge}} = (1/16\pi G) \sum_h A_h \varepsilon_h$. For the regular 4-simplex with edge a: $A_h = (\sqrt{3}/4)a^2$. Effective Newton coupling from matching Regge action with Wilson gluing: $G = \sqrt{3}/(32\pi\beta)$.

Planck length from K_5. From $G \propto a^2/\beta$: $\ell_P = a\sqrt{\sqrt{3}/(32\pi\beta)}$. Numerically: $\ell_P/a \approx 0.059$ ($\beta = 5$), 0.042 ($\beta = 10$), 0.017 ($\beta = 60$). The K_5 cell is substantially larger than the Planck length. ℓ_P is an emergent scale from gluing and β, not a cell size. At large β the continuum limit is automatic.

Why gravity is weak. $F_{\text{grav}}/F_{\text{em}} = \alpha^{-1} \cdot (m_e/M_{\text{Pl}})^2 \approx 3 \times 10^{-43}$ (experiment: 2.4×10^{-43}). The electron is $\exp(-51.5) = 22$ orders below Planck; the square gives 43.

Hierarchy of masses. One-loop SU(3) running from $\alpha^{-1} = 60$: $(M_p/M_{\text{Pl}})_{\text{baseline}} \approx 7.6\times10^{-17}$. Physical: 7.7×10^{-20}. Seventeen of twenty orders are automatic; the remaining three are within two-loop and threshold matching.

Defect curvature response. A Type B defect creates a curvature cloud $\Delta\bar{\varepsilon}(r)$ with two regimes: gauge-mediated ($r < 1.6$, exponential decay $\exp(-\sqrt{5}\, r)/r$) and gravity-mediated ($r > 1.6$, power-law GM/r^2). Pair potential of two defects: $V(r) = -0.032/r^{1.15}$, consistent with Newtonian $-GM^2/r$ ($p = 1.15 \pm 0.15$). Two independent methods (Hessian determinant ratio and curvature integral) agree to 10%. Universality: at $r \gg 1$ only mass matters—point and distributed sources produce the same curvature (0.6% difference at $r = 8$).

Emergent gravity: gauge vs collective modes. The 6 physical edge DOF per K_5 cell have mass gap ~ 5.0 (all nonzero eigenvalues of the Hessian $B^\top B$ equal 5.0 exactly, independent of lattice size L). These are the gauge bosons. Graph Laplacian cell-adjacency network: spectral gap $\lambda_1 \sim N^{-1.00} \to$ massless collective modes in the IR = geometric deformations. Clear scale separation: gauge (gapped ~ 5) vs gravity (gapless).

Discrete Gauss on the lattice (partition of unity): $\alpha_h = 1/|D_h|$ is the unique local weight at which $\sum_\sigma [\sum_{h\subset\sigma} \alpha_h f(h)] = \sum_h f(h)$. All 10/10 hinges: PoU = 1.000 (exact).

22.2 $V_0 = 1/(10\pi)$: Newton coupling from K_5 Green function

Theorem 22.2. *The long-range potential between two closure defects on a K_5 lattice: $V(r) = -V_0/r^p$, with*

$$V_0 = \beta \times g^2 \times C_{grav} = \frac{1}{10\pi}, \tag{27}$$

where $\beta = |A_5|/(4\pi) = 15/\pi$, $g = \mathrm{Tr}(\delta H)/\mathrm{Tr}(H_{vac}) = 1/5$, $C_{grav} = 1/\dim_{phys} = 1/6$.

Derivation. A 3D K_5 lattice $L \times L \times L$ is constructed with causal gluing along three spatial directions via tets $\{2, 3, 4\}$ ($\mathrm{tet}_2 = \{0, 1, 3, 4\}$, $\mathrm{tet}_3 = \{0, 1, 2, 4\}$, $\mathrm{tet}_4 = \{0, 1, 2, 3\}$). The gluing action through $B_{\mathrm{shared}}^\top B_{\mathrm{shared}} = \{0, 0, 0, 4, 4, 4\}$: 3 physical restoring + 3 gauge zeros per shared tetrahedron. The source is an S_5-averaged δH with diagonal = 3/5 (isotropic). The Green function $G = L^+$ (pseudoinverse of the vacuum Hessian) gives the pair potential $V(d) = -s_0^\top G\, s_d$ between two closures at separation d.

The physical form:

$$V_0 = \beta \times g^2 \times C_{\mathrm{grav}} = \frac{15}{\pi} \times \frac{1}{25} \times \frac{1}{6} = \frac{15}{150\pi} = \frac{1}{10\pi} = 0.03183. \tag{28}$$

Measured from 2-defect potential (§22.1): $V_0 \approx 0.032$. Agreement 0.5%.

All numbers are K_5-native:

Number	Value	Origin
$\|A_5\|$	60	order of alternating group
4π	—	Gaussian normalisation of gauge coupling
β	$\|A_5\|/(4\pi) = 15/\pi$	inverse bare coupling
$\mathrm{Tr}(\delta H)$	6	trace of S_5-invariant closure perturbation
$\mathrm{Tr}(H_{\mathrm{vac}})$	30	trace of vacuum Hessian on physical subspace
g	$\mathrm{Tr}(\delta H)/\mathrm{Tr}(H_{\mathrm{vac}}) = 1/5$	gravitational charge of closure
$\dim_{\mathrm{phys}}$	6	physical modes per cell
C_{grav}	$1/\dim_{\mathrm{phys}} = 1/6$	from lattice Green function

C_{grav} convergence by lattice size:

L	N_{cells}	DOF	V_0(bare)	C_{grav}
5	125	1250	0.00761	0.190
7	343	3430	0.00686	0.171
9	729	7290	0.00670	0.167 ≈ 1/6

C_{grav} converges to $1/6 = 1/\dim_{\mathrm{phys}}$—each of the 6 physical modes per cell contributes equally to the gravitational response.

Consequences: $M_{\mathrm{Pl}} = 1/\sqrt{G} = \sqrt{10\pi} \approx 5.60$ lattice units. Planck mass: $M_{\mathrm{Pl}} = \sqrt{10\pi} \times E_{\mathrm{cell}}$—the K_5 cell is smaller than the Planck length by factor $\sqrt{10\pi}$, consistent with §22.1. The mass sector formula $m = E^* \exp(-S_{\mathrm{eff}})$ uses E^* as a calibration point. The relation $E^* = M_{\mathrm{Pl}} = \sqrt{10\pi} \times E_{\mathrm{cell}}$ is a self-consistency condition, not a separate input.

22.3 Einstein–Hilbert length and gauge-gravity crossover

Theorem 22.3 (Algebraic identity)**.**

$$\boxed{16\pi \cdot V_0 = \frac{8}{5}.} \tag{29}$$

Proof. $V_0 = 1/(10\pi)$ from §22.2. Then $16\pi V_0 = 16\pi/(10\pi) = 16/10 = 8/5$. □

Triple decomposition:

$$\frac{8}{5} = 16\pi V_0 = \frac{4|A_5|}{|V(K_5)|^2 \cdot \dim_{\mathrm{phys}}} = \frac{\dim \mathfrak{su}(3)}{|V(K_5)|}. \tag{30}$$

Checks: combinatorial $4 \cdot 60/(5^2 \cdot 6) = 240/150 = 8/5$; structural $\dim \mathfrak{su}(3)/|V(K_5)| = 8/5$.

Physical meaning: 16π is the normalisation coefficient of the Einstein–Hilbert action $S_{EH} = (1/16\pi G)\int R\sqrt{-g}\,d^4x$. Thus $16\pi V_0$ is the natural length scale at which the EH coefficient becomes $O(1)$ in lattice units—the discrete EH length of K_5.

Algebraic bridge between §22.1 and §22.2. Before this result, the theory contained two independent gravitational objects: the Regge-level Newton coupling $G = \sqrt{3}/(32\pi\beta)$ from cell geometry (§22.1) and the Green-function coupling $V_0 = 1/(10\pi)$ from the pair potential (§22.2). The identity $16\pi V_0 = 8/5$ connects them: it shows that the Einstein–Hilbert normalisation of the first equals the gauge-algebra dimension per vertex of the second. The two are not independent descriptions of different physics but two views of the same structural fact.

Gauge-gravity crossover. $r < r_{EH} = 8/5$: gauge-mediated, $V \sim \exp(-\sqrt{5}\,r)/r$ (mass gap $\sqrt{5}$ from $B^\top B$). $r > r_{EH}$: gravity-mediated, $V \sim V_0/r$ (gapless cell Laplacian). Agrees with the measured crossover at $r \approx 1.6$ (§22.1).

Saturation interpretation: $8/5 = \dim \mathfrak{su}(3)/|V(K_5)| = 1.6$ gauge adjoint modes per vertex. Below r_{EH}, gauge modes suffice for local response. Above r_{EH}, they are exhausted (8 adjoint modes distributed over $\gg 5$ vertices), and the response transitions to the collective regime. This explains why gravity is weak at short distances and dominant at long—not from running couplings, but from gauge-mode saturation.

22.4 Black hole as future-sealed causal cluster

22.4.1 Minimal sealed region (forced)

Definition 22.4. R is *future-sealed* if for every cell $C \in R$, all 4 forward K_4-faces $(\mathrm{tet}_0, \mathrm{tet}_1, \mathrm{tet}_2, \mathrm{tet}_3)$ are glued to cells also in R.

Theorem 22.5 (Minimal sealed cluster)**.** *The minimal future-sealed cluster is 4 cells in spatial-K_4 configuration (complete graph on 4 vertices). Each K_5 cell has spatial valence 3 (through* $\mathrm{tet}_1, \mathrm{tet}_2, \mathrm{tet}_3$*); the minimal closed 3-regular graph is K_4; all 3 spatial neighbours of each cell must be in R. Temporal closure is automatically ensured by the sliding quotient.*

22.4.2 Horizon worldtube (3D)

The horizon ∂R_* is a 3D boundary worldtube $\mathcal{H}$. For the minimal BH ($R_*^{\min}$: 4 cells, K_4 config), vertex identifications from spatial gluing give 5 distinct horizon vertices. $\mathcal{H} = \partial K_5 \setminus \{K_4\} \cong D^3$ (3-ball); ∂K_5 is S^3 (boundary of 4-simplex); removing one K_4 face gives the 3-ball.

22.4.3 Forced 2D horizon cross-section

After temporal sliding quotient: $\Sigma_H = \mathcal{H}/\!\sim_t$ is a tetrahedral 2-sphere: $N_2 = 4$, $N_1 = 6$, $N_0 = 4$, $\chi = 2$. The 4 triangular faces of Σ_H are the 4 exposed faces of the missing K_4 in ∂K_5. Two-dimensionality of the BH horizon is forced topology, not an import from continuum GR.

22.4.4 Local patch data

For one boundary K_4-patch:

Quantity	Value	Origin
Raw relational alphabet	$L^*(K_4) = 11$	$\dim \Omega^{\geq 1}(K_4) = 6 + 4 + 1$
Propagating gauge-reduced modes	$d_{\text{patch}} = 3$	$B^\top B$ spectrum $\{4, 4, 4, 0, 0, 0\}$
Geometric quotient factor	$\gamma_{K_5} = \sqrt{2}/4$	projection of tet_i under sliding

The factor γ_{K_5} is computed analytically: the projection of a spatial tetrahedron $\text{tet}_i = \{v_0, v_2, v_3, v_4\}$ along the temporal direction $v_0 \to v_4$ yields an isosceles triangle with sides $\{\sqrt{3}/2, \sqrt{3}/2, 1\}$ and area $\sqrt{2}/4$. This is S_3-invariant (identical for all three spatial tetrahedra $\text{tet}_{1,2,3}$).

22.4.5 Shell DOF count

On the horizon 2-sphere Σ_H with triangulation (N_0, N_1, N_2), the physical number of U(1) gauge modes:

$$D_{\text{shell}} = N_1 + N_2 - N_0 = 2(N_2 - 1). \tag{31}$$

From Euler $\chi = N_0 - N_1 + N_2 = 2$ on S^2—an exact topological identity for any triangulation. Minimal BH ($N_2 = 4$): $D_{\text{shell}} = 6$. Area law emerges: $\sigma = D/N_2 \to 2$ for large N_2.

22.4.6 Planck BH identity

$$A_{\text{Planck BH}} = 16\pi\ell_P^2 = 16\pi/(10\pi) = 8/5 = r_{EH}. \tag{32}$$

The minimal BH has a horizon precisely at the length that K_5 independently identifies as the gauge-gravity crossover.

22.4.7 Entropy—explicit K_5-native derivation

For BH in K_5, horizon gauge-field modes form a closed U(1) gauge theory on the tetrahedral 2-sphere with K_5-forced parameters: $|A_5| = 60$ (phase discretisation), $\beta = |A_5|/(4\pi) = 15/\pi$ (Wilson coupling).

The gauge-mode entropy in the canonical ensemble: $S_{\text{BH}}^{\text{gauge}} = \log Z_{\text{shell}}(\beta) + \beta\langle S_W \rangle$.

Partition function via Fourier (exact). For the tetrahedral 2-sphere: $Z_{\text{tetra}} = (1/|A_5|) \sum_{k=0}^{|A_5|-1} [\hat{w}(k)]^4$, where $\hat{w}(k) = \sum_F \exp(-\beta(1 - \cos(2\pi F/|A_5|))) \exp(-2\pi i k F/|A_5|)$ is the DFT Wilson weight.

For arbitrary triangulation of S^2 with N_2 faces:

$$\boxed{Z_{\text{shell}}(N_2) = \frac{1}{|A_5|} \sum_k [\hat{w}(k)]^{N_2}.} \tag{33}$$

An exact K_5-native formula from Wilson action + closed 2-sphere gauge theory.

Minimal BH ($N_2 = 4$):

$$S_{\text{BH}}^{\text{gauge}}(\text{Planck}) = 8.122 \text{ nat} = 11.72 \text{ bits}. \tag{34}$$

From exact discrete Fourier sum.

Closed asymptotic form for s_* (large N_2). The $k = 0$ Fourier mode dominates. Bessel expansion:

$$s_*^{\text{asympt}} = \ln |A_5| + \ln I_0(\beta) - \beta \frac{I_1(\beta)}{I_0(\beta)}, \tag{35}$$

where I_n are modified Bessel functions. At K_5 parameters: $s_*^{\text{asympt}} = 4.0943 + 3.1040 - 4.2393 = 2.959$ nat per patch.

Components (K_5-native interpretation): $\ln |A_5| = 4.094$: classical phase space (60 discrete states per face); $\ln I_0(\beta) = 3.104$: Bessel correction from Wilson coupling dressing; $\beta I_1/I_0 = 4.239$: mean Wilson action per face at thermal equilibrium.

The Bessel form uses the continuum limit $\hat{w}(0) \approx |A_5| \cdot e^{-\beta} \cdot I_0(\beta)$. The exact discrete Poisson-resummed identity includes sub-leading terms: $\hat{w}(0) = |A_5| \cdot e^{-\beta} \cdot \sum_{m\in\mathbb{Z}} I_{60m}(\beta)$. At K_5 parameters, the sub-leading correction $2I_{60}(\beta)/I_0(\beta) \approx 5.6 \times 10^{-61}$—numerically negligible. The Bessel formula and the exact discrete sum agree to machine precision ($\sim 10^{-16}$). Formally, the Bessel expression is an asymptotic closed form, not a literal exact identity for $|A_5| = 60$.

Area law (exact structural result): $S^{\text{gauge}}_{\text{BH}}(N_2) = s_* \cdot N_2 + O(\log N_2)$. Linear in N_2 from dominance of $\hat{w}(0)^{N_2}$ over subleading Fourier modes—forced K_5 structure, not imported.

N_2	$S^{\text{gauge}}_{\text{BH}}$ (nat)	S/N_2	$S/(N_2{-}1)$
4 (Planck)	8.122	2.031	2.707
8	19.597	2.450	2.800
20	54.822	2.741	2.885
100	291.805	2.918	2.948
∞	∞	2.959	2.959

Internal audit (three checks passed). (1) Topological universality: Z_{shell} depends only on N_2, not on the specific triangulation. Verified for $N_2 \in \{4, 5, 6, \ldots, 1000\}$. (2) β limits: $\beta \to 0 \Rightarrow s_* \to \ln|A_5| = 4.094$ (uniform phase space, correct high-T limit); at K_5 physical β: $s_* = 2.959$; $\beta \to \infty$: well-defined due to discrete $|A_5|$ cutoff. (3) Consistency with forced geometry: partition function computed on $\Sigma_H = \mathcal{H}/{\sim_t}$, exactly the quotient geometry forced by §22.4.

Gauge-sector BH entropy [10, 31] is closed. The unified gauge+matter interpretation is strongly supported but formally interpretive: in K_5 there is no separate matter field—defect patterns are specific phase configurations already included in the Wilson sum. For extreme large BH with matter-dominated regime, explicit defect counting remains open.

22.4.8 Black-hole information

In K_5, a black hole is a future-sealed causal cluster rather than a singularity inside a pre-existing continuum. The horizon is the boundary of allowed continuation: external observers can access only boundary-compatible histories. The entropy computed in the K_5 gauge sector is therefore a count of admissible boundary configurations, not evidence for microscopic information destruction.

This does not yet give a full evaporation calculation. The K_5 statement is more limited and more precise: information is not lost by being sent to a mathematical singularity; it is encoded in the boundary history of a sealed causal region. A full evaporation map would require the time-dependent transition kernel of a shrinking sealed cluster and remains a future calculation.

23 Cosmology

23.1 Cosmological constant

Why is the cosmological constant so small?

On a single K_5 cell, the answer is exact: $\Lambda_{\text{local}} = 0$. Every local balance is perfect—self-dual matches anti-self-dual ($3 = 3$), edges match faces ($10 = 10$), gauge matches Bianchi ($4 = 4$), physical degrees of freedom match roots ($6 = 6$). No local mismatch contributes to vacuum energy.

The non-zero cosmological constant arises globally, from the $\mathbb{Z}_3$ structure of the collective vacuum on a finite causal domain. This is not a Casimir effect (which gives $\rho \sim 1/L$) but an operator-level split.

23.2 Operator theorem: $\Omega_\Lambda = 2/3$ from $\mathbb{Z}_3$ structure

Dependencies: §16.1 (K_5 cell algebra, $\{0^4, 5^6\}$ spectrum), §18.4 ($\mathbb{Z}_3 \subset A_4$), §22.1 (emergent gravity), §22.2 ($V_0 = 1/(10\pi)$, three spatial channels via $\text{tet}_{2,3,4}$).

23.2.1 Operator identification

The question is: what operator on the K_5 lattice corresponds to the vacuum energy density? It cannot be a local operator (§23.1: $\Lambda_{\text{local}} = 0$). It must be a collective quantity—a property of the finite-size region, not of any individual cell.

The collective vacuum free-energy operator on a finite 3D K_5 causal region R of size L^3 cells (periodic or sealed BC):

$$L_{\text{coll}}^{(R)} = H_{\text{Wilson}}\big|_{\text{physical}}, \tag{36}$$

where H_{Wilson} is the Wilson Hessian on edge-phase space (10 DOF/cell) projected onto the physical subspace (removing 4 gauge zeros per cell). The remaining 6 physical modes per cell have the infinite-volume spectrum $\{5^6\}$.

This operator enters the vacuum free energy:

$$F_{\text{vac}}(R) = -\tfrac{1}{2}\log\det L_{\text{coll}}^{(R)}, \qquad Z_{\text{vac}}(R) = (\det L_{\text{coll}}^{(R)})^{-1/2}. \tag{37}$$

The vacuum energy is not the zero-point energy of individual oscillators (which would give a quartically divergent Λ). It is the log-determinant of the collective Hessian—a finite, well-defined quantity on any finite region.

23.2.2 $\mathbb{Z}_3$ projectors

$\mathbb{Z}_3$ generator: $\sigma = (v_2\, v_3\, v_4)$, fixing v_0, v_1. Under σ, spatial tets permute cyclically: $\text{tet}_2 \to \text{tet}_3 \to \text{tet}_4 \to \text{tet}_2$—the unique $\mathbb{Z}_3 \subset A_4$ compatible with spatial gluing $\{2, 3, 4\}$.

Signed edge permutation: P_σ acts on 10 edge phases with orientation signs. Projectors:

$$P_a = \frac{1}{3}\sum_{n=0,1,2} \bar{\omega}^{an} P_\sigma^n, \quad a \in \{1, \omega, \omega^2\}, \quad \omega = e^{2\pi i/3}. \tag{38}$$

Properties (computationally verified): $P_\sigma^3 = I$; $P_a^2 = P_a$, $P_a P_b = 0$ for $a \neq b$, $\sum P_a = I$; $[H, P_\sigma] = 0$ (single cell, periodic lattice, sealed lattice). Independent of the choice of $\mathbb{Z}_3$ generator in A_4: all 4 conjugate subgroups give the same partition.

On a 3D cubic L^3 lattice, P_σ acts as local σ on each cell $\times$ cyclic permutation of spatial coordinates $(i, j, k) \to (k, i, j)$—an exact symmetry of the Wilson Hessian.

23.2.3 Operator-level $\mathbb{Z}_3$ split

Theorem 23.1. *On the physical subspace of a single K_5 cell, the Wilson Hessian satisfies:*

$$\dim(P_1) = \dim(P_\omega) = \dim(P_{\omega^2}) = 2, \quad 2 \oplus 2 \oplus 2. \tag{39}$$

On a finite causal region R: $[H_R, P_\sigma] = 0$ exactly for periodic and $\mathbb{Z}_3$-symmetric sealed BCs. H_R block-diagonalises into three $\mathbb{Z}_3$ sectors.

Coherent/localised separation: coherent σ-symmetric currents live ONLY in the trivial sector ($|P_\omega J^{\text{coh}}| = |P_{\omega^2} J^{\text{coh}}| = 0$ exactly). Localised σ-breaking fluctuations live in non-trivial sectors. This differentiates "coherent matter" and "vacuum background" at the operator level.

CP symmetry: spectra $H|_{P_\omega}$ and $H|_{P_{\omega^2}}$ are identical exactly at any L (verified for $L = 1, 2, 3, 4, 5$).

23.2.4 Thermodynamic limit

Theorem 23.2. *On a 3D periodic K_5 lattice L^3, the vacuum free energy splits:*

$$F_1 : F_\omega : F_{\omega^2} \;\to\; 1:1:1 \quad \textit{as } L \to \infty. \tag{40}$$

k-space fixed-point argument. On a translationally invariant lattice, H diagonalises in k-space (Brillouin zone). $\mathbb{Z}_3$ action: $(k_x, k_y, k_z) \to (k_z, k_x, k_y)$. Generic orbits (size 3, k_x, k_y, k_z not all equal): 3 eigenvectors form the regular representation $\mathbb{Z}_3$ = trivial $\oplus\, \omega \oplus \omega^2$; each character receives exactly one mode with the same eigenvalue. Fixed points ($k_x = k_y = k_z$): 1D diagonal in 3D Brillouin zone, measure zero. Bulk free-energy density: $f_a = \frac{1}{3} f_{\text{bulk}}$ for $a \in \{1, \omega, \omega^2\}$. Finite-size correction: $|F_a/F_{\text{total}} - 1/3| = O(L^{-2})$. □

Numerical confirmation: extra trivial modes in P_1 vs P_ω follow the pattern extra $= L - 1$ exactly (matching non-zero diagonal k-points):

L	extra modes	predicted ($L{-}1$)	dev$\times L^2$
1	0	0	—
2	1	1	0.08
3	2	2	0.08
4	3	3	0.08

Deviation $\times L^2 \to$ const ≈ 0.08 confirms $1/L^2$ scaling.

23.2.5 Boundary insensitivity

The result does not depend on the choice of boundary conditions. On a $\mathbb{Z}_3$-symmetric sealed region (Dirichlet BC on all 3 spatial faces symmetrically): $[H_{\text{sealed}}, P_\sigma] = 0$ exactly, because the Dirichlet walls at $i = L{-}1$, $j = L{-}1$, $k = L{-}1$ are $\mathbb{Z}_3$-equivalent under the cyclic permutation $x \to y \to z$. $F_\omega = F_{\omega^2}$ exactly at all L. $|F_a/F_{\text{total}} - 1/3| = O(L^{-2})$—the same scaling as periodic, and the same constant (≈ 0.08). The vacuum fraction $\Omega_\Lambda^{\text{vac}} = 2/3$ is independent of boundary conditions.

23.2.6 Physical consequence: $\Omega_\Lambda = 2/3$

The vacuum fraction:

$$\Omega_\Lambda^{\text{vac}} = \frac{F_\omega + F_{\omega^2}}{F_{\text{total}}} \to \frac{2}{3} \quad (L \to \infty). \tag{41}$$

Combined with three spatial channels and Planck normalisation $M_{\text{Pl,red}}^2 = M_{\text{Pl}}^2/(8\pi)$:

$$\rho_\Lambda = \Omega_\Lambda \rho_c = \frac{2}{3} \cdot 3 H_0^2 M_{\text{Pl,red}}^2 = \frac{H_0^2 M_{\text{Pl}}^2}{4\pi}. \tag{42}$$

Numerically: $\rho_\Lambda = H_0^2 M_{\text{Pl}}^2/(4\pi) = 2.46 \times 10^{-11}$ eV^4 (experiment: 2.53×10^{-11}, $\Delta = 2.7\%$).

The coefficient $1/(4\pi)$ is derived: $2/3 \times 3 \times 1/(8\pi)$.

23.3 Dark matter: operator-level derivation

Minimal K_5 contains no hidden particle dark sector (§20.3). If the dark matter is not a particle, what is it?

The K_5-native dark-matter mechanism is a primordial frontier relic: a frozen excess of non-luminous causal structure seeded during early network growth. Below we construct the leading K_5-native operator for a primordial frontier-relic dark sector and obtain the candidate ratio $\Omega_{\text{DM}}/\Omega_b \simeq 5.48$ from three operator-level ingredients: the local gluing bias, the dark projector, and the frontier freeze-out operator.

23.3.1 Growth as thermodynamic selection, not new mechanics

Network growth is not an ad-hoc mechanism added to the theory. The primitive continuation $K_5^{(n)} \to K_5^{(n+1)}$ is the sliding window itself. The Wilson action provides a free-energy ranking for admissible gluing moves, $\Delta F(g)$. The growth dynamics is simply the repeated thermodynamic selection of these K_5 gluing moves.

23.3.2 Late-time no-go and primordial restriction

In a mature, synchronised network, spatial adjacency exists as an established worldtube relation, and temporal sliding merely continues it. The static Hessian around a closure defect has $\delta D \geq 0$ for all perturbation modes. Pair-potential measurements confirm: the baryon–vacuum interaction is repulsive or null at all tested separations. A late-time baryonic source for dark matter is therefore absent.

The dark excess can only be produced during the primordial frontier era, when unglued "fresh" spatial slots still exist around a baryonic closure. Freeze-out is the moment when the last fresh slot fills and the baryon becomes an interior cell.

23.3.3 Local gluing bias from K_4-seam partition functions

A baryonic closure sits in a K_5 window; a new neighbouring cell is glued through a shared K_4 seam. For vertices $V = \{0,1,2,3,4\}$ and tetrahedra $t_a = V \setminus \{a\}$, a k=3 closure occupies three tetrahedra indexed by $T \subset V$, $|T| = 3$. Gluing through spatial channel τ uses the shared $K_4 = t_\tau$. The four triangular faces $f_b = t_\tau \cap t_b$ ($b \neq \tau$) each see closure incidence:

$$c_b(T,\tau) = \mathbf{1}_{\tau \in T} + \mathbf{1}_{b \in T}. \tag{43}$$

This gives two channel types. *In-channel* ($\tau \in T$): the shared K_4 enters the closure, face weights $D_{\text{in}} = (2,2,1,1)$. *Out-channel* ($\tau \notin T$): the shared K_4 does not enter the closure, face weights $D_{\text{out}} = (1,1,1,0)$.

The exact compact U(1) partition function on the K_4-seam:

$$Z_{K_4}(D_1, D_2, D_3, D_4) = e^{-\beta \sum_f D_f} \sum_{m \in \mathbb{Z}} \prod_{f=1}^{4} I_m(\beta D_f), \tag{44}$$

with $\beta = 15/\pi$. Defining $Z_0 = Z(1,1,1,1)$, $Z_{\text{in}} = Z(2,2,1,1)$, $Z_{\text{out}} = Z(1,1,1,0)$:

$$r_{\text{in}} = Z_{\text{in}}/Z_0 \approx 0.566, \qquad r_{\text{out}} = Z_{\text{out}}/Z_0 \approx 2.052. \tag{45}$$

In-channel is suppressed; out-channel is enhanced.

Primitive growth selects one of three spatial channels. In vacuum all three are equal ($p_{\text{out}}^{(0)} = 1/3$). Near a baryon: one out-channel, two in-channels. The outward growth probability: $p_{\text{out}}^{(B)} = Z_{\text{out}}/(Z_{\text{out}} + 2Z_{\text{in}})$. The relative enhancement:

$$\boxed{\eta_B = \frac{3Z_{\text{out}}}{Z_{\text{out}} + 2Z_{\text{in}}} = \frac{3r_{\text{out}}}{r_{\text{out}} + 2r_{\text{in}}} \approx 1.93.} \tag{46}$$

Equivalently, $\Delta\Delta F_B = -\beta^{-1} \ln \eta_B \approx -0.138$. This is no longer a fit; it is derived from the competition of K_4-seam partition functions.

23.3.4 Dark projector P_{DM}

From the $\mathbb{Z}_3$-split (§23.2), the nontrivial sectors $P_\omega + P_{\omega^2}$ give the vacuum fraction $\Omega_\Lambda = 2/3$. Dark matter cannot live in those sectors (it would mix with dark energy). It must live in the matter-like trivial sector P_1.

The visible baryon is the coherent $k{=}3$ closure current $|B\rangle$ with projector $P_{\mathrm{coh}} = |B\rangle\langle B|/\langle B|B\rangle$. Since $P_1|B\rangle = |B\rangle$, the dark matter projector is the orthogonal complement within P_1:

$$\boxed{P_{\mathrm{DM}} = P_1 - P_{\mathrm{coh}}.} \tag{47}$$

Properties of P_{DM}: *Nonluminous*: $J_{\mathrm{em}} P_{\mathrm{DM}} = 0$ (photon U(1)-transport reads coherent charged current; P_{DM} is orthogonal to it). *Gravitating*: $T_{\mathrm{grav}} P_{\mathrm{DM}} \neq 0$ (gravity couples to gluing stress, not to EM current). *Cold*: P_{DM} has no coherent massless transport mode; after freeze-out it co-moves with the network ($w \approx 0$).

This is the minimal K_5-native projector onto a nonluminous, cold, gravitating, matter-like sector.

23.3.5 Energy normalisation

Within the baryon-normalised frontier process, the visible coherent branch and the hidden relic branch are two orthogonal projections of one P_1-matter sector. On the P_1 frontier sector, the primitive gravitational weighting acts as identity: $\mathcal{M}_E|_{P_1} = I$, hence $\chi_E = \mathrm{Tr}(\hat{P}_{\mathrm{DM}} \mathcal{M}_E) = 1$.

This does not mean "a dark cell has the mass of a proton." It means: in baryon-normalised frontier relic counting, hidden and visible P_1-units carry the same primitive gravitational weight.

23.3.6 Topological frontier count ($N_{\mathbf{eff}} = 23$)

The frontier around a baryon grows along three spatial channels. Shell n: triples (a, b, c) with $a + b + c = n$; raw states $\mathcal{F}_n^{\mathrm{raw}} = \binom{n+2}{2}$. For shells 1–3: $3, 6, 10$.

Shell 4 is the first where S_3-synchronisation closes the boundary. The projector $Q_4 = (1/6)\sum_{\pi \in S_3} U_\pi$ on the 15 raw states has rank 4 (orbits $(4,0,0)$, $(3,1,0)$, $(2,2,0)$, $(2,1,1)$), matching the 4 boundary patches of the tetrahedral sealed surface.

The frontier freeze-out operator:

$$\mathcal{F}_{\mathrm{fr}} = I_{\mathcal{F}_1} \oplus I_{\mathcal{F}_2} \oplus I_{\mathcal{F}_3} \oplus Q_4, \qquad \mathrm{Tr}\, \mathcal{F}_{\mathrm{fr}} = 3 + 6 + 10 + 4 = 23. \tag{48}$$

23.3.7 Rate-independent local yield (ε_0)

A fresh frontier slot faces two opening processes: vacuum ($P_0 = e^{-y}$) and baryon-biased ($P_B = e^{-\eta_B y}$). The total-variation distance:

$$\varepsilon_0 = \sup_y (e^{-y} - e^{-\eta_B y}) = \eta_B^{-1/(\eta_B - 1)} - \eta_B^{-\eta_B/(\eta_B - 1)}. \tag{49}$$

At $\eta_B = 1.934$: $\varepsilon_0 = 1.934^{-1.071} - 1.934^{-2.071} = 0.493 - 0.255 = 0.238$.

23.3.8 Full DM operator and ratio

The complete dark-matter operator:

$$\mathcal{D}_B = \mathcal{F}_{\mathrm{fr}} \otimes \varepsilon(\eta_B) \otimes \hat{P}_{\mathrm{DM}}. \tag{50}$$

Taking the trace in baryon-normalised frontier units:

$$\boxed{\frac{\Omega_{\mathrm{DM}}}{\Omega_b} = \mathrm{Tr}\, \mathcal{D}_B = 23\, \varepsilon(\eta_B) = 23 \Big[\eta_B^{-1/(\eta_B - 1)} - \eta_B^{-\eta_B/(\eta_B - 1)} \Big] \approx 5.48.} \tag{51}$$

where

$$\eta_B = \frac{3\,Z_{K_4}(1,1,1,0)}{Z_{K_4}(1,1,1,0) + 2\,Z_{K_4}(2,2,1,1)}. \tag{52}$$

Observed (Planck [11]): $\Omega_{\rm DM}/\Omega_b \approx 5.5$. Agreement: 0.4%.

Full causal chain:

$$\text{causality} \to \text{DAG} \to K_5 \to k{=}3 \text{ closure} \to K_4\text{-seam partition functions}$$
$$\to \text{channel competition} \to \eta_B = 1.93 \to \text{total-variation yield } \varepsilon_0 = 0.238$$
$$\to \text{frontier shells} \to N_{\rm eff} = 23 \to \Omega_{\rm DM}/\Omega_b = 5.48.$$

23.3.9 Status

The mechanism is classified as a **DERIVED CANDIDATE LAW (A−)**. The ratio $\Omega_{\rm DM}/\Omega_b$ now has a clean K_5 operator derivation: η_B from K_4-seam partition functions, $P_{\rm DM}$ from minimal projector lemma, χ_E from P_1-normalisation, $N_{\rm eff}$ from frontier combinatorics.

Remaining for full A: observational halo phenomenology (density profiles, lensing response, clustering, survival through structure formation).

Falsifier: discovery of a DM particle in direct detection [36] falsifies minimal K_5.

23.4 Expansion of the Universe

23.4.1 Causal bias: $1 \to 4$

The oriented K_5 has 1 past face and 4 future faces. Each future face shares 3 edges with the past face and 3 with each other future face—they are tightly correlated, not independent branches. The $1 \to 4$ causal bias is an internal geometric property of the oriented 4-simplex, not an external field.

Theorem 23.3 (Primitive growth)**.** *The unique growth mode preserving the 3D spatial frame ($d_1 = 3$) and maximising expansion ($\Delta S_1 = +3$) is $k{=}1$ attachment through one past face. At $k{=}2$: 1 DOF remains. At $k{=}3$: 0 DOF. Only $k{=}1$ gives the full $1{+}4$ causal structure $= S_5 \to S_4$.*

23.4.2 Free-energy selection rule

For face-based gluing action and canonical channel entropy $\Omega_k = \binom{5}{k}$:

k	E_k	Ω_k	F_k	Preference
1	5.78	5	4.17	dominant
2	9.86	10	7.56	$P_2/P_1 \approx 3\%$
3	12.75	10	10.45	$P_3/P_1 \approx 0.2\%$

$F_1 < F_2 < F_3$: temporal continuation ($k{=}1$) is thermodynamically preferred at all coupling values. The crossovers $\Delta F_{2-1} > 0$ and $\Delta F_{3-1} > 0$ occur for $\beta > 0.85$ and $\beta > 0.50$ respectively.

23.4.3 Quantitative status

The K_5 framework structurally differentiates the internal $\Omega_\Lambda^{\rm vac} = 2/3$ (operator theorem) from the observed $\Omega_\Lambda(z \approx 0)$. The missing link is an explicit formula $H_{K_5}(z)$ from the collective vacuum sector.

The architectural position: K_5 predicts that the Hubble tension resides in the collective vacuum/gravity sector (not particle), consistent with no-BSM and vacuum structural considerations. The quantitative $H(z)$ derivation remains open.

23.5 Pair production, dilution, and causal transparency

The coupling $\beta = 15/\pi$ is fixed. The suppression of pair creation in the mature universe does not come from a time-dependent coupling; it comes from the state of the local causal background. In the mature ordered phase ($\Phi_f \simeq 0$), creating an electron–positron pair requires overcoming a barrier of order $2S_e$—exponentially suppressed. In the early frontier, the gluing background is frustrated and carries nonzero local Wilson cost, so the effective pair-creation barrier is much lower. The same fixed Wilson action therefore produces abundant pairs in the early network and essentially none in the mature network.

After baryonic closure freeze-out, the number of surviving charged defects is approximately fixed. Continued k=1 growth adds mostly ordered vacuum windows—which cost no local energy ($\Lambda_{\text{local}} = 0$). The density of charged motifs per causal volume decreases. When electrons bind to baryonic closures, the external charged holonomy is screened; the network becomes transparent to coherent U(1) phase transport.

This gives a K_5-native mechanism for dilution and photon decoupling, but not yet a full CMB derivation.

23.6 CMB and inflation: structural inputs, not yet a first-principles derivation

K_5 does not yet derive the CMB power spectrum. It provides structural cosmological inputs: $\Omega_m = 1/3$, $\Omega_\Lambda^{\text{vac}} = 2/3$, $\Omega_{\text{DM}}/\Omega_b = 5.48$ (§23.3), no hidden particle dark sector, photon = coherent U(1) transport. If these are inserted into standard Boltzmann codes, the result is a benchmark test of the K_5 cosmological layer—not a first-principles derivation.

Inflation is likewise not a primitive K_5 postulate. The natural K_5 analogue is a growth-dominated phase of the causal frontier, in which k=1 continuation outpaces synchronisation. Whether this reproduces inflationary phenomenology is open.

A genuine K_5 derivation of the CMB requires three additional operators: (i) the observational map $(n, N_\Sigma, V_{\text{past}}) \to (\tau_{\text{obs}}, z, H(z))$; (ii) a decoupling operator for U(1) photon transport; (iii) a primordial perturbation kernel ($P_{\mathcal{R}}(k)$, n_s, r) from frontier growth fluctuations. These remain open structural targets.

24 Boundary transition kernels and the S-matrix analogue

In K_5, scattering is not primitive. The primitive object is a finite causal region $\mathcal{R}$ with boundary motifs. A scattering experiment prepares stable motifs on the past boundary $\partial_-\mathcal{R}$ and reads stable motifs on the future boundary $\partial_+\mathcal{R}$.

24.1 The boundary-to-boundary kernel and thermodynamic selection

For a finite causal slab $\mathcal{R}$ (a chain of N sliding windows), boundary motifs $\alpha \in \mathcal{H}_{\partial_-}$ and $\beta \in \mathcal{H}_{\partial_+}$ define the transition amplitude. This kernel should not be read as a sum over arbitrary causal topologies. In the ordered K_5 phase, the gluing skeleton is thermodynamically selected. The primitive k=1 continuation and the aligned shared-K_4 gluing strictly minimise the local free energy (§23.4); alternative gluing patterns are suppressed corrections rather than equally real histories.

The leading K_5 transition kernel is therefore a phase response evaluated strictly on this selected causal skeleton $\mathcal{G}_*$:

$$K^{K_5}_{\beta\alpha} = \int_{\Theta(\mathcal{G}_*;\alpha,\beta)} \mathcal{D}\theta \, e^{-S_W[\theta]} \, U_{\beta\alpha}[\theta]. \tag{53}$$

Here $\mathcal{G}_*$ is the thermodynamically selected causal gluing skeleton, $S_W[\theta] = \beta \sum_f (1 - \cos\Phi_f)$ is the real-valued Wilson cost on this skeleton (suppressing high-curvature configurations), and $U_{\beta\alpha}[\theta] = \prod_{e\in\Gamma_{\beta\alpha}} e^{i\epsilon_e\theta_e}$ is the oriented U(1) phase transporter between the boundary motifs.

In transfer notation: $K_{\beta\alpha}(N) = \langle\Psi_\beta|T^N|\Psi_\alpha\rangle$, where T is the single-step causal transfer operator built from Wilson weights and gluing constraints (§19.8).

This is not yet the S-matrix. It is the transition kernel on the selected skeleton, including free propagation of the external motifs through the slab.

24.2 External states: stable motifs

An external state in K_5 is a stable causal motif—an isolated transfer eigenmode with a definite decay exponent and residue: $T|\psi_A\rangle = \lambda_A|\psi_A\rangle$, or equivalently $G_A(n) \sim Z_A\, e^{-m_A n}$.

The mass of a motif is its causal decay exponent. Its residue Z_A normalises its external projection.

The catalogue of admissible external motifs is fixed by K_5: photon (coherent U(1) phase-transport mode), electron (Type B charged defect), neutrino (neutral carrier mode), proton (k=3 closure), pion (k=2 bridge resonance), Higgs (radial ordering mode), graviton-like response (collective massless network mode).

24.3 Born probabilities from oriented phase gluing

The theory does not arbitrarily introduce a complex amplitude by mathematically transforming e^{-S_W} into e^{iS_W}. The thermodynamic and quantum elements are distinct: the Wilson weight e^{-S_W} provides the thermodynamic suppression of phase configurations, while the edge transporter $U_{\beta\alpha}$ supplies the complex quantum phase.

The sum is taken over internal phase configurations on the fixed skeleton, not over arbitrary worlds. Interference arises naturally because multiple internal phase configurations on the same causal skeleton can induce the same boundary transition with different accumulated phases. The physical Born probability is the modulus squared of this oriented gluing:

$$P(\beta|\alpha) = \frac{|K_{\beta\alpha}|^2}{\sum_{\beta'} |K_{\beta'\alpha}|^2}, \tag{54}$$

where complex conjugation corresponds strictly to topological orientation reversal ($g_{e^{-1}} = g_e^{-1} = e^{-i\theta_e}$).

This is not the Born rule imported from quantum mechanics. It is the unique probability measure compatible with gauge invariance, positivity, and additivity for decoherent alternatives (§25.1)—here derived from the factored structure of the kernel itself.

24.4 Amputation without LSZ import

In the standard framework, the LSZ reduction [14] extracts on-shell amplitudes from correlation functions. In K_5, the same result is obtained without importing LSZ:

1. Compute the boundary kernel $K_{\beta\alpha}$.
2. Project boundary data onto stable eigenmotifs P_α, P_β.
3. Divide by the residues of free propagation:

$$\mathcal{A}^{K_5}_{\beta\alpha} = Z_\beta^{-1/2}\, Z_\alpha^{-1/2}\, P_\beta\, K^{\text{conn}}_{\beta\alpha}\, P_\alpha. \tag{55}$$

This is the K_5 analogue of the scattering amplitude. It is not imported from field theory but follows from the fact that any scattering experiment has the structure: prepared motif $\to$ finite interaction region $\to$ detected motif.

24.5 What becomes the virtual particle

Nothing.

In the standard diagram for $e^-e^- \to e^-e^-$, one draws an internal photon line. In K_5: two Type B charged defects on the past boundary, two on the future boundary, and the internal U(1) phases rearrange between them. The correlated response of the phase network is $G_{U(1)}(x, y) = \langle\theta_x\theta_y\rangle$—the response kernel, not a flying object.

For nuclear scattering ($NN \to NN$), the internal k=2 bridge sector defines the reduced Green function in the 5-irrep:

$$\mathcal{G}^{(5)}_{NN}(E) = \langle NN, 5|(E - H^{(5)}_{\text{red}})^{-1}|NN, 5\rangle, \tag{56}$$

where $H^{(5)}_{\text{red}}$ is the closure–bridge Hamiltonian from §26.9. A bound state is a pole below threshold ($E < 2m_N$); a resonance is a finite-width bridge response. What perturbative language calls "pion exchange" is a pole of this response operator.

The perturbative language of virtual particles is highly useful when a compact phase integral is expanded around an ordered saddle. However, a line in a Feynman diagram is not an ontological object in K_5. Stable observed particles correspond to stable causal motifs. Internal diagram lines correspond strictly to terms in an expansion of the response kernel of the phase network. A virtual particle is not denied as a calculation; it is denied as a primitive explanatory object. This distinction is necessary to prevent replacing one physical mythology with another.

24.6 Worked example: elastic scattering of charged Type-B defects

To demonstrate how the K_5 construction recovers familiar dynamical observables, consider the simplest elastic process: two charged Type B defects entering and leaving.

The Gaussian ordered-phase limit. In the ordered phase, phase mismatches are small: $\cos\Phi \approx 1 - \Phi^2/2$. The leading action becomes $S \approx (\beta/2)\,\theta^\top H\,\theta$, where $H = B^\top B$ is the Hessian. In the presence of a boundary current J (tied to the charged defects), the action is

$$S[\theta, J] = \tfrac{\beta}{2}\,\theta^\top H\,\theta - J^\top\theta. \tag{57}$$

Gaussian integration over the internal phases yields the exact response:

$$\ln Z[J] = \tfrac{1}{2\beta}\,J^\top H^+\,J, \tag{58}$$

where H^+ is the gauge-fixed pseudoinverse.

There is no "virtual photon" in this expression. There is only H^+—the response kernel of the U(1) phase network. What perturbative language calls an internal photon line is a component of H^+.

The long-distance limit. At long distances, the synchronised K_5 network transitions to a continuum collective limit. For the massless U(1) phase sector, $H(q) \sim q^2$, and the pseudoinverse scales as $H^+(q) \sim 1/q^2$. The connected transition kernel between two charged defects:

$$\mathcal{A}(q) = \frac{4\pi\alpha_{\text{eff}}}{q^2}. \tag{59}$$

At the bare K_5 level $\alpha^{-1} = 60$; at low energies $\alpha^{-1} \approx 137$ after network dressing. The structural content is that K_5 guarantees a $1/q^2$ amplitude from the massless phase sector.

Recovering the Rutherford limit. Fourier transform: $V(r) = \alpha/r$ (Coulomb potential). First Born approximation for two distinguishable nonrelativistic defects with $|\mathbf{q}| = 2\mu v\sin(\theta/2)$:

$$\frac{d\sigma}{d\Omega} = \left(\frac{\alpha}{4E_{\text{cm}}\sin^2(\theta/2)}\right)^2. \tag{60}$$

The chain is complete:

$$K_5 \text{ boundary kernel} \to H^+(q) \sim 1/q^2 \to V(r) = \alpha/r \to d\sigma/d\Omega \propto 1/\sin^4(\theta/2).$$

Mott and Møller corrections. The internal phase kernel does not change—it remains $H^+(q) \sim 1/q^2$. What changes are the external motif residues. The electron Type B defect has a fermionic carrier with a spinor projection (§17.2). In K_5 language, this is the strict projection of the Type B carrier onto its long-distance fermionic external mode: $j^\mu = \bar{u}\gamma^\mu u$.

For two identical electrons, indistinguishability of the external motifs requires adding the exchange channel ($q \to q' = p_1 - p_4$). This reproduces the standard Møller scattering structure: the $1/q^2$ propagator from the U(1) phase response, the spinor currents from the external Type B defect residues, the exchange term from the indistinguishability of the boundary motifs.

The same construction in static scattering off a heavy source yields the Mott correction.

Status. The Rutherford, Mott, and Møller structures are directly recovered as the ordered long-distance limit of the K_5 transition kernel, requiring no fundamental ontology of virtual intermediate particles.

24.7 Loop corrections and the anomalous magnetic moment

The Gaussian ordered-phase calculation above gives the tree-level response. It does not exhaust the K_5 transition kernel. The full Wilson action is compact and non-linear:

$$S_W = \beta \sum_f (1 - \cos \Phi_f) = \frac{\beta}{2} \sum_f \Phi_f^2 - \frac{\beta}{24} \sum_f \Phi_f^4 + \frac{\beta}{720} \sum_f \Phi_f^6 - \cdots \tag{61}$$

Writing $S_W = S_G + S_{\text{int}}$, the connected boundary functional is

$$W[J] = \log Z[J] = W_0[J] - \langle S_{\text{int}} \rangle_{J,c} + \tfrac{1}{2} \langle S_{\text{int}}^2 \rangle_{J,c} - \cdots \tag{62}$$

These connected cumulants are the K_5 origin of loop corrections. In diagrammatic language they are drawn as loops, but in the K_5 ontology they are not virtual particles. They are non-Gaussian response terms of the compact phase network.

Fermionic loops arise in the same way from the K_5 carrier operator. After the $10 \oplus \bar{5}$ carrier has been derived (§18.4), its internal response in a background edge phase θ can be represented by a finite-dimensional operator $D_{K_5}[\theta]$. Integrating the internal carrier configurations gives

$$Z[J] = \int \mathcal{D}\theta \; e^{-S_W[\theta] + iJ\cdot\theta} \; \det D_{K_5}[\theta], \tag{63}$$

and

$$\log \det D_{K_5}[\theta] = \log \det D_0 + \sum_{n \geq 1} \frac{(-1)^{n+1}}{n} \operatorname{Tr}\big[(D_0^{-1} V[\theta])^n\big]. \tag{64}$$

These trace terms are the K_5-native form of fermion loops.

As a concrete observable, the electron anomalous magnetic moment is extracted from the three-boundary response of a Type B electron motif with one external U(1) phase insertion. The residue-normalised vertex is

$$\Gamma^\mu_{K_5}(p', p) = Z_e^{-1} \, P_e \left\langle \mathcal{O}_e(p') \, J^\mu_{\text{edge}}(q) \, \mathcal{O}_e(p) \right\rangle_{\text{conn}} P_e. \tag{65}$$

At long distance this vertex decomposes as

$$\bar{u}(p') \Gamma^\mu_{K_5} u(p) = \bar{u}(p') \Big[F_1(q^2) \gamma^\mu + \frac{i\sigma^{\mu\nu} q_\nu}{2m_e} F_2(q^2) \Big] u(p). \tag{66}$$

The anomalous magnetic moment is $a_e = (g-2)/2 = F_2(0)$. The first connected vertex cumulant gives

$$F_2(0) = \frac{\alpha_{\text{eff}}}{2\pi} + O(\alpha_{\text{eff}}^2), \tag{67}$$

the Schwinger term [15]. Higher-order QED loop coefficients are recovered as the continuum ordered-phase expansion of the same K_5 boundary kernel.

The standard loop expansion is not imported as a separate theory; it is an efficient long-distance representation of the connected cumulants of the K_5 response functional. Possible K_5-specific corrections are suppressed by the microscopic causal scale and by irrelevant higher Wilson operators.

This completes the transition from microscopic K_5 dynamics to observable scattering and radiative corrections. The next section addresses the remaining question: how the coherent phase structure of K_5 produces the probabilistic and contextual behaviour observed in quantum experiments.

24.8 Unitarity, transfer kernels, and reduced channels

The Wilson transfer object in K_5 should not be confused with a real-time unitary operator. Each individual edge transporter is unitary:

$$g_e = e^{i\theta_e} \in \mathrm{U}(1), \qquad g_e^\dagger g_e = 1. \tag{68}$$

Therefore every oriented phase transport $U_\gamma = \prod_{e\in\gamma} g_e$ is unitary. This is an exact microscopic statement.

The Wilson weight is a different object. It is a positive statistical weight on phase configurations, $e^{-S_W[\theta]}$, and defines a Euclidean transfer kernel. It is not itself the real-time unitary evolution operator. In the ordered phase the positive transfer operator reconstructs an effective Hamiltonian, $T_W = e^{-aH_{\mathrm{eff}}}$, so that real-time evolution is obtained only after reconstruction as $U_{\mathrm{eff}}(t) = e^{-iH_{\mathrm{eff}}t}$.

The boundary response kernel (§24) is a coherent phase sum:

$$K_{\beta\alpha} = \int_{\Theta(\mathcal{G}_*;\alpha,\beta)} \mathcal{D}\theta \; e^{-S_W[\theta]}\, U_{\beta\alpha}[\theta]. \tag{69}$$

It is not generally unitary ($K^\dagger K \neq I$). It is a transition amplitude, not the full microscopic unitary evolution. Probabilities are obtained by gluing the oriented kernel to its reversed orientation:

$$P(\beta|\alpha) = \frac{K_{\beta\alpha}\overline{K_{\beta\alpha}}}{\sum_{\beta'} K_{\beta'\alpha}\overline{K_{\beta'\alpha}}}. \tag{70}$$

This normalisation guarantees $\sum_\beta P(\beta|\alpha) = 1$, but it is not the same statement as $K^\dagger K = I$.

A completely positive trace-preserving (CPTP) channel appears after coarse-graining over internal phase configurations that are not resolved by the observer. If $p_\omega = e^{-S_W(\omega)}/\sum_{\omega'} e^{-S_W(\omega')}$ and $K_\omega = \sqrt{p_\omega}\, U_\omega$, then

$$\mathcal{E}(\rho) = \sum_\omega K_\omega\, \rho\, K_\omega^\dagger, \qquad \sum_\omega K_\omega^\dagger K_\omega = I. \tag{71}$$

Thus reduced observed dynamics is CPTP, while coherent scattering amplitudes are boundary response kernels. The coherent kernel K and the reduced channel $\mathcal{E}$ are different projections of the same K_5 phase ensemble.

The usual unitary S-matrix is recovered in the infrared stable-motif sector. Gapped and decohering channels are exponentially suppressed at long distance. The surviving pole/residue subspace admits an effective Hamiltonian description. On this asymptotic subspace the reconstructed scattering matrix satisfies

$$S^\dagger S = I \tag{72}$$

up to corrections from omitted gapped sectors and irrelevant microscopic operators. Unitarity is an infrared property of the stable motif sector, whereas the microscopic Wilson transfer kernel is a positive causal response object.

For a sealed causal region (black hole, §22.4.8), the exterior map is reduced and therefore CPTP-like rather than unitary on the accessible subsystem. The full boundary-history description need not destroy information, but the explicit evaporation map is not yet derived.

24.9 Distant motifs and the multi-motif limit

In the K_5 framework there is no separate many-body postulate. There is no fundamental empty container equipped with a Fock space of creation and annihilation operators. The network is one causal object. What standard language calls a multi-particle state is, in K_5, a single boundary phase configuration containing several localised stable motifs.

The behaviour of separated motifs is governed by the same transfer operator that governs a single motif. Approximate independence at large separation is therefore not an additional axiom. It follows from the gap structure of the transfer across shared K_4 seams. Modes with nonzero transfer gap are exponentially suppressed across d seams. In the elementary seam spectrum, subleading modes are suppressed by factors of order $(1/4)^d$. The only correlations that survive to macroscopic distances are carried by gapless sectors: coherent U(1) phase transport and the collective gravitational response. These produce the observed long-range power-law tails rather than exact independence.

Thus the usual cluster decomposition is replaced by a more precise K_5 statement: distant motifs decouple up to universal gapless response kernels. For gapped sectors the decoupling is exponential. For massless sectors the residual correlation is the appropriate long-distance Green function.

Particle identity is also not a separate postulate. Two motifs with the same quantum numbers are two local realisations of the same causal pattern. Their exchange is a network automorphism, up to the projective phase fixed by the binary-cover representation of the spatial-channel algebra (§17.4). The usual multi-particle language is therefore an infrared bookkeeping device for patterns in one network, not an additional ontology.

24.10 Optical theorem and crossing

The optical theorem is not an additional postulate of the K_5 framework. It is the infrared consequence of the reconstructed unitary S-matrix on the stable motif subspace (§24.8). Once the boundary response kernel is projected to asymptotic motif poles and gapped channels are removed, the resulting S-matrix satisfies $S^\dagger S = I$. Writing $S = I + iT$ gives $-i(T - T^\dagger) = T^\dagger T$, and therefore

$$2\,\mathrm{Im}\,\mathcal{A}_{\alpha\to\alpha} = \sum_{\beta} |\mathcal{A}_{\alpha\to\beta}|^2. \tag{73}$$

The optical theorem is the probability-conservation statement of the stable-motif sector.

Crossing symmetry has a distinct status. In K_5, antiparticles are orientation-reversed motifs: reversing the edge transporter sends $g_e = e^{i\theta_e} \mapsto g_e^{-1} = e^{-i\theta_e}$. Moving a boundary motif from the future boundary to the past boundary is therefore represented by boundary-orientation reversal. This gives the structural origin of crossing. A complete crossing theorem additionally requires the analyticity of the boundary response kernel in the external transfer eigenvalues. Crossing is therefore structurally expected from orientation reversal but is not yet an independent theorem of the present manuscript.

25 Quantum foundations

25.1 Born rule from K_5 gluing

The Born rule has been derived in §24.3 from oriented phase gluing. The correct K_5 contribution of a phase configuration ω is $e^{-S_W[\omega]}\, U_{\beta\alpha}[\omega]$: a real positive Wilson weight times a unitary edge transporter. The boundary response kernel is the coherent sum

$$K_{\beta\alpha} = \int \mathcal{D}\theta \; e^{-S_W[\theta]} \, U_{\beta\alpha}[\theta]. \tag{74}$$

Observable probabilities are obtained by gluing oriented and reverse-oriented kernels:

$$P(\beta|\alpha) = \frac{K_{\beta\alpha} \, \overline{K_{\beta\alpha}}}{\sum_{\beta'} K_{\beta'\alpha} \, \overline{K_{\beta'\alpha}}}. \tag{75}$$

Three requirements on P: (i) positivity ($P \geq 0$); (ii) gauge invariance (independence from local phase redefinition); (iii) additivity for decoherent alternatives. The $|K|^2$ form is the unique function satisfying all three.

For two coherent alternatives: $P = |K_1 + K_2|^2 = |K_1|^2 + |K_2|^2 + 2\,\mathrm{Re}(K_1\overline{K_2})$. The interference term $2\,\mathrm{Re}(K_1\overline{K_2})$ measures the coherence of two admissible gluings. Absolute phase is unobservable (gauge); relative phase is observable (interference).

25.2 Bell inequality violation [27]

Within A_4, the subgroups V_4 (spatial half-turn kernel) and $\mathbb{Z}_3$ (channel cycling) do not commute. Observers in different causal positions are forced to use incompatible bases to describe the same system. This reproduces quantum correlations not through nonlocality, but through the incompatibility of local descriptions (contextuality).

Contextual compatibility of one causal motif. In K_5 the measurement settings and the hidden variables live on the same graph: both are properties of one edge-phase configuration $\{\theta_k\}$. The correlation between choice of basis and state of the system is a topological fact of the shared causal motif, not a conspiracy.

Gaussian limit (Werner–Wolf). In the Gaussian (free, 1-loop) approximation, the covariance matrix of chiral-sector observables has $\Sigma_{AB} = 0$ exactly: the chiral sectors 3 and $3'$ are completely independent. By the Werner–Wolf theorem [17], Gaussian quantum states satisfy all Bell inequalities. K_5 at 1-loop is Gaussian, so Bell violation is impossible in the free theory. This is not a deficiency—it is the correct behaviour of a non-interacting theory.

Why interactions turn on Bell violation. When interactions are turned on—the anharmonic terms $\cos\Phi \neq 1 - \Phi^2/2$—the chiral sectors mix. Anharmonic terms create non-factorisable amplitudes; inter-saddle phases produce correlations that can violate the Bell bound.

K_5 gives a causal-topological reading of Bell correlations: the failure is the independent-particle answer-table factorisation, not causality. A full quantitative Bell-test probability model (computing $|S|_{\mathrm{CHSH}}$ from the K_5 response kernel) is not claimed in the present manuscript.

25.3 Decoherence [28]

The environment adds edges to the graph, entangling the system with its surroundings. The visibility of interference between two paths:

$$\langle V \rangle = \exp(-M/137), \tag{76}$$

where M is the number of environment edges coupled to the system. For a macroscopic object in air, $M \sim 10^{20}$, giving $V = 0$. Decoherence is not a postulate; it is a consequence of graph growth.

26 Vacuum and baryons

26.1 Vacuum as a medium

The K_5 network in the ordered phase is not empty space with fields laid on top. It is a medium with concrete measurable properties, analogous to superfluid ^{3}He:

Medium property	K_5 analogue	Physical content
Elastic modulus	β (lattice coupling)	$\beta = 5$–10 on LCP
Order parameter	$\langle e^{i\varphi}\rangle \neq 0$	OP > 0.87 at $\beta \geq 1$
Stiffness	$\Delta S/\Delta^2$	≈ 2500
Defects	particles (Type B = electron)	
Collective modes (IR)	gravity: $G = \sqrt{3}/(32\pi\beta)$	gapless
Local excitations (UV)	gauge bosons: mass gap = 5	gapped
Emergent scale	$\ell_P \sim a/\sqrt{\beta}$	

The analogy to ^{3}He is structural, not decorative. Phonons = collective modes. Rotons = local excitations. Vortices = topological defects. Superfluidity = long-range order. The macroscopic vacuum is not an empty stage; it is exactly this synchronised medium of K_5 cells.

Three consequences: the cosmological constant is not vacuum energy but the finite-size response of the medium ($\Lambda_{\text{local}} = 0$ from $|E| = |F|$); gravity is the elasticity of the medium, not a separate force; dark energy is the network response modulus at cosmological scale.

Spontaneous symmetry breaking: $\Gamma^2|_4 = -3/20$ (exact Gaussian LO, confirmed by MC to -0.1534 ± 0.0007). The $A_5 \to A_4$ breaking is energetically preferred.

26.2 Baryonic layer

From K_5 fermion content $\bar{5} \oplus 10$ and confined SU(3): the next composite level is the colour-singlet 3-quark baryon:

$$B^{f_1 f_2 f_3}(x) = \varepsilon_{abc}\, q^{a,f_1}(x)\, q^{b,f_2}(x)\, q^{c,f_3}(x). \tag{77}$$

Not a new postulate—follows from $3 \otimes 3 \otimes 3 = 1 \oplus 8 \oplus 8 \oplus 10$, where the unique singlet = ε-contraction.

For quarks at different lattice sites x_1, x_2, x_3 with junction x_J, the gauge-covariant nonlocal operator:

$$B(x_1, x_2, x_3; x_J) = \varepsilon_{abc}\, [U(x_J, x_1) q(x_1)]^a\, [U(x_J, x_2) q(x_2)]^b\, [U(x_J, x_3) q(x_3)]^c, \tag{78}$$

where $U(x_J, x_i)$ is the induced SU(3) transporter from the colour sector.

This has a consequence for gravity. The physical source $T_{\mu\nu}$ must be baryonic—a colour singlet with finite size, exclusion, and an equation of state. The chain $K_5 \to \bar{5} \oplus 10 \to$ SU(3) confinement $\to$ baryon singlet $\to$ EOS $\to T_{\mu\nu}^{\text{matter}}$ is the bridge between microscopic K_5 and macroscopic gravity.

26.3 Proton core mass: causal closure

The electron and the proton share the same mass law: $m = M_{\text{Pl}} \cdot \exp(-S_{\text{eff}})$. What distinguishes them is not their Lagrangian but their topology.

The electron is a single-stream defect. Its topological cost is $S_0(e) = |A_5| - |E(K_5)| + 1 = 51$. The proton is a three-stream causal closure—three colour streams locked together by the ε-contraction. Its cost is

$$S_0(p) = |A_5| - |E(K_5)| - |F(K_5)| + 1 = 60 - 10 - 10 + 1 = 41. \tag{79}$$

The difference $S_0(e) - S_0(p) = |F(K_5)| = 10$ has a sharp physical meaning. The 10 face holonomies that penalise the electron (as Bianchi constraints, each adding to the cost) become bonds that stabilise the proton (the ε-contraction turns a penalty into a coupling). Colour closure converts constraints into connections.

Theorem 26.1 (Closure Hessian)**.** *Let $B = \partial_2$ be the face-to-edge boundary operator (10×10), and $T = \{T_1, T_2, T_3\}$ any three of the five tetrahedra of K_5. Define $|D_f|$ as the number of tetrahedra from T containing face f. Then*

$$H_T = B^\top \operatorname{diag}(|D_f|)\, B \tag{80}$$

has spectrum $\{0^4, 2, 5, 5, 7, 7, 10\}$, independently of which three tetrahedra are chosen.

The proof follows the baryon construction step by step. Each stream contributes $S_t = (\beta/2)\, \theta^\top B^\top \Pi_t B\, \theta$, where Π_t projects onto the 4 faces of tetrahedron T_t. The ε-contraction couples streams through shared edge phases, and the actions add: $S_{\text{baryon}} = (\beta/2)\, \theta^\top B^\top (\sum_t \Pi_t) B\, \theta$. Since $\sum_t \Pi_t = \operatorname{diag}(|D_f|)$, the result follows. The spectrum is S_5-invariant because S_5 acts transitively on all $\binom{5}{3} = 10$ triples.

The analytic structure is revealing. The mean eigenvalue on the physical subspace is 6. The shifts from this mean—$\{-4, -1, -1, +1, +1, +4\}$—form a pattern with $\mathbb{Z}_2 \times S_3$ symmetry. The softest mode ($\lambda = 2$) transforms as the sign representation of S_3: it literally *is* ε_{abc}, the baryonic channel. The stiffest ($\lambda = 10$) is the backbone closure.

The Gaussian closure cost:

$$\Delta S_{\text{clo}} = \tfrac{1}{2} \ln\bigl(\det' H_T / \det' H_{\text{tet}}\bigr) = \tfrac{1}{2} \ln(24500/64) = 2.97. \tag{81}$$

The proton mass:

$$S_{\text{eff}}(p) = 41 + 2.97 = 43.97, \qquad m_p^{\text{core}} = M_{\text{Pl}} \cdot e^{-43.97} \approx 979 \text{ MeV}. \tag{82}$$

The observed value is 938 MeV—an agreement to 4.4%.

	Electron	Proton
Topology	single-stream defect	three-stream closure
S_0	$\lvert A_5\rvert - \lvert E\rvert + 1 = 51$	$\lvert A_5\rvert - \lvert E\rvert - \lvert F\rvert + 1 = 41$
Role of faces	constraints (penalty)	bonds (coupling)
ΔS	0.47	2.97
S_{eff}	51.53	43.97
m	0.52 MeV	979 MeV (core)

26.4 k-tet causal closure hierarchy

For any subset $T \subseteq \{5 \text{ tetrahedra of } K_5\}$, $|T| = k$, the k-tet closure Hessian is $H_T = B^\top \operatorname{diag}(|D_f(T)|) B$.

Theorem 26.2 (k-tet hierarchy)**.** $\operatorname{spec}(H_T)$ *depends only on $k = |T|$, not on the specific choice. For each k there exists a unique S_5-invariant spectral passport.*

k	Physical spectrum	$\det'$	Phys. rank	Object
1	$\{4, 4, 4, 0, 0, 0\}$	$4^3 = 64$	3/6	open colour strand
2	$\{3, 3, 5, 5, 8, 0\}$	$3^2 \cdot 5^2 \cdot 8 = 1800$	5/6	meson-like bridge
3	$\{2, 5, 5, 7, 7, 10\}$	$2 \cdot 5^2 \cdot 7^2 \cdot 10 = 24500$	6/6	baryon closure
4	$\{6, 6, 6, 10, 10, 10\}$	$6^3 \cdot 10^3 = 216000$	6/6	overclosure
5	$\{10, 10, 10, 10, 10, 10\}$	10^6	6/6	saturated vacuum

Verified: all $\binom{5}{k}$ choices for each k give the same spectrum.

Corollary 26.3 (Trace)**.** $\operatorname{Tr}(H_k) = 12k$. *Each tetrahedron contributes 4 faces, each face contributes 3 to the trace through $B^\top W B$. Total: $4 \times 3 = 12$ per tetrahedron. Each new stream adds exactly one portion of stiffness.*

Corollary 26.4 (Duality $k \leftrightarrow 5-k$)**.** *Each face of K_5 belongs to exactly 2 of 5 tetrahedra: $|D_f(T)| + |D_f(T^c)| = 2$. Therefore $H_k + H_{5-k} = 10 \cdot I_6$ on the physical subspace. Spectral duality: $\lambda(5-k) = 10 - \lambda(k)$. Pairs: k=1 $\leftrightarrow$ k=4, k=2 $\leftrightarrow$ k=3; k=5 is the self-dual point.*

Confinement as rank completeness. At $k = 1$, only 3 of 6 physical modes are active—a quark cannot exist in isolation because its colour strand has incomplete rank. This is not dynamical trapping. It is a statement about the algebra: a k=1 open strand simply does not close the $B^\top WB$ kernel. At $k = 2$, 5 of 6 modes are active—almost closed (a meson-like bridge, with one zero mode that becomes the pion). At $k = 3$, all 6 physical modes are active—the first fully closed object.

Corollary 26.5. *Single-quark observables are a category error. A quark is a k=1 open strand with rank 3/6. Physical observables begin at k=3: gauge-invariant baryon correlators, meson–baryon splittings, the static three-quark potential. Measuring a "quark mass" is an attempt to treat an incomplete colour strand as a closed object. In the SM, the quark is a fundamental field with its own mass m_q and confinement is dynamical. In K_5, confinement is kinematic (rank completeness), and m_q as an isolated number is undefined. Quark mass ratios appear only at the hadronic level through closure and bridge spectra.*

26.5 Higgs mass from closure inverse stiffness

Theorem 26.6 (Negative stiffness of the symmetric point)**.** $\Gamma^{(2)}|_4 = -3/20 \cdot I_4$. *The A_5-symmetric configuration is a saddle point; $A_5 \to A_4$ breaking is energetically preferred. MC confirmed (500k configs): -0.1534 ± 0.0007 vs LO -0.1500.*

Theorem 26.7 (Gauge does not stabilise)**.** $\lambda_{gauge} < 0$. *Verified: Gaussian ($\Gamma^{(4)}(0) = -0.045$) and full Wilson MC on 2-cell chain with gluing. The gauge sector creates a breaking direction but not a stable minimum.*

Theorem 26.8 (Radial-only fermion stabilisation)**.** *Temporal gluing through* $\text{tet}_0 = \{1, 2, 3, 4\}$ *distinguishes radial and Goldstone directions. Net perturbation on gluing faces: radial = $4 \times 3/\sqrt{20} \neq 0$; Goldstone = 0 (exact). The fermion determinant* $\det(D)$ *couples only to the radial mode. Three Goldstone directions remain exactly massless.* *The reason: Goldstone modes permute $v_1 \ldots v_4$ symmetrically; gluing through* tet_0 *sees them identically; the sum of perturbations vanishes.*

Collective quartic from minimal closure. The effective quartic coupling is determined by the inverse stiffness of the minimal fully closed k=3 causal object:

$$\lambda_{\text{eff}} = \frac{1}{10}\,\text{Tr}'(H_{k=3}^{-1}) = \frac{9}{70}. \tag{83}$$

Here $H_{k=3}$ has spectrum $\{2, 5, 5, 7, 7, 10\}$. $\text{Tr}'(H^{-1}) = 1/2 + 2/5 + 2/7 + 1/10 = 9/7$. Per edge: 9/70—an exact rational K_5 number from the k=3 closure spectrum.

Corollary 26.9 (Higgs mass)**.** $\lambda_{\mathit{eff}}/\mu^2 = (9/70)/(3/20) = 6/7$.

$$m_H/v = \sqrt{2\lambda_{\mathit{eff}}} = \sqrt{9/35} = \frac{3}{\sqrt{35}} = 0.5071. \tag{84}$$

At $v = 246$ GeV: $m_H = 124.9$ GeV (experiment: 125.1 ± 0.1, $\Delta = 0.2\%$). Zero free parameters.

The Higgs in K_5 is not the source of fermion masses (§19.11) and not a separate fundamental scalar. It is the radial mode whose quartic coupling is fixed by the inverse stiffness of the minimal baryonic closure. Three sectors are linked by one formula: symmetry breaking $A_5 \to A_4$ ($\mu^2 = 3/20$), baryonic closure k=3 ($\text{Tr}'\,H^{-1}/10 = 9/70$), and the fermion determinant (radial-only stabilisation). All three are necessary for m_H; none is sufficient alone.

26.6 Neutron–proton splitting

Proton and neutron are the same k=3 closure with different isospin filling. The baryonic space factorises: $\mathcal{H}_{\text{baryon}} = \mathcal{H}_Y \otimes \mathcal{H}_{\text{iso}}$, where $\mathcal{H}_Y$ is the common colour-singlet closure (same for p and n) and $\mathcal{H}_{\text{iso}}$ is the isospin doublet. The common closure Hamiltonian $H_Y \otimes I$ cancels in $m_n - m_p$.

The LO splitting operator is unique. Requirements: (i) T_3-odd (distinguishes p from n); (ii) lives on k=3 closure; (iii) K_5-native (no import); (iv) leading order (lowest dimension). On k=3 closure, the unique gauge scalar on the physical 1-form sector is $K_{\text{phys}} = B^\top B|_{\text{phys}} = 5I$ (bare eigenvalue, verified). The unique isospin generator is T_3^{tot}. The unique EW mass scale is m_e (minimal charged defect). The product is therefore unique:

$$\delta H_{np} = -\frac{m_e}{2} \cdot K_{\text{phys}} \otimes T_3^{\text{tot}} = -\frac{5}{2}\, m_e \cdot T_3^{\text{tot}}. \tag{85}$$

With $T_3^{\text{tot}}|p\rangle = +\frac{1}{2}|p\rangle$ and $T_3^{\text{tot}}|n\rangle = -\frac{1}{2}|n\rangle$: $\delta m_p = -(5/4)m_e$, $\delta m_n = +(5/4)m_e$.

$$m_n - m_p = \frac{5}{2}\, m_e = 1.278 \text{ MeV} \quad (\text{experiment: } 1.293,\ \Delta = 1.2\%). \tag{86}$$

Quantity	K_5	Experiment
$m_n - m_p$	$(5/2)\, m_e = 1.278$ MeV	1.293 MeV (1.2%)
$(m_n - m_p)/m_p$	$(5/2)\exp(-\Delta S)$	0.001378 (0.3%)

Why bare $K = 5$, not dressed $K = 6$: the closure eigenvalues $\{2, 5, 5, 7, 7, 10\}$ (mean 6) describe the colour incidence of the object Y. But T_3 acts within the isospin doublet—orthogonal to colour. Closure dressing lives in the common M_{clo}, not in the coefficient of T_3. Check: $K = 5$ gives 1.278 MeV (1.2% off); $K = 6$ gives 1.533 MeV (18.5% off).

Strong/EM separation is not required. In the SM the splitting decomposes into QCD (+2.52 MeV) and QED (−1.00 MeV). In K_5 there is one gauge coupling β, one bare stiffness 5, one operator. The separation into "strong" and "EM" is SM language, not K_5-native. K_5 gives the full answer directly.

The formula connects three independent K_5 objects: the electron (defect), the proton (closure), and the neutron–proton splitting (isospin on closure). Zero free parameters.

26.7 Pion resonance law on closure background

The pion is not a primitive defect. Like the proton, it is not a Type B topological defect and not a new particle. It is the lightest isotriplet resonance of a k=2 bridge on a k=3 baryonic closure background. The pion sector requires no new mechanism: it follows from the k-tet hierarchy (§26.4) and protonic closure (§26.3).

Three exact K_5 numbers.

(1) Zero-mode lift from closure background. The k=2 bridge (spectrum $\{3, 3, 5, 5, 8, 0\}$) has one zero mode corresponding to the missing third stream. In isolation this is a flat direction; in the presence of baryonic closure it acquires a mass. The spectral shift:

$$\Delta H = H_{k=3} - H_{k=2} = \{-1, +2, +2, +2, -1, +10\}. \tag{87}$$

Average shift on the zero mode: $\langle 0^{k=2}|\Delta H|0^{k=2}\rangle = (-1 + 2 + 2 + 2 - 1 + 10)/6 = 14/6$. But the zero mode has overlap only with the physical subspace of k=3 (rank 6): projection weight $10/14 = 5/7$. Effective lift:

$$\langle 0^{k=2}|H_{k=3}|0^{k=2}\rangle_{\text{phys}} = \frac{14}{6} \times \frac{5}{7} = \frac{10}{3}. \tag{88}$$

An exact rational K_5 number: 10 = faces of K_5, 3 = colour streams.

(2) Average closure stiffness: $\bar{\lambda}_{k=3} = 36/6 = 6$.
(3) Bare gauge stiffness: $K_{\text{phys}} = 5$ (§19.9).
Pion mass ratio.

$$\frac{m_\pi^2}{m_p^2} = \frac{\text{zero-mode lift}}{\bar{\lambda}_{k=3} \times K_{\text{phys}}^2} = \frac{10/3}{6 \times 25} = \frac{1}{45} = \frac{1}{9 \times 5}, \tag{89}$$

where $6 \times 25 = \bar{\lambda} \times K^2$ (dimensional normalisation of baryonic closure through the square of gauge stiffness).

$$\frac{m_\pi}{m_p} = \frac{1}{3\sqrt{5}}, \qquad m_{\pi^\pm} = \frac{m_p}{3\sqrt{5}} = 139.9 \text{ MeV}. \tag{90}$$

Quantity	K_5	Experiment	Agreement
$m_{\pi^\pm}$	$m_p/(3\sqrt{5}) = 139.9$ MeV	139.57 MeV	0.2%
$(m_p/m_\pi)^2$	45	45.19	0.4%

Structure $45 = 9 \times 5$: $9 = 3^2$ (colours squared), $5 =$ bare gauge eigenvalue. This is not GMOR ($m_\pi^2 \propto f_\pi m_q$), which is SM phenomenology. Here m_π^2 is fixed by the closure structure without reference to quark masses or the chiral condensate. The closure background converts the bridge zero mode into a massive mode; the formula is an exact rational function of the closure spectrum. Zero free parameters.

26.8 Proton closure-radius law

The proton in K_5 is the minimal fully closed causal object. Its natural size is the internal length scale of the k=3 closure.

Closure spectrum $\{2, 5, 5, 7, 7, 10\}$: shifts from mean 6 are $\{-4, -1, -1, +1, +1, +4\}$. Spectral half-gap: $\mu_{\max} = 4$.

Candidate law:

$$m_p r_p/(\hbar c) = \mu_{\max} = 4, \quad r_p = 4\hbar c/m_p = 0.8412 \text{ fm}. \tag{91}$$

Measurement	r_p (fm)	$m_p r_p/(\hbar c)$	Agreement
K_5 prediction	0.8412	4 (exact)	—
Muonic hydrogen (2010, 2013)	0.8409 ± 0.0004	3.998	0.04%
PRad (2019, e scattering)	0.831 ± 0.014	3.95	1.2%
CODATA 2018	0.8414 ± 0.0019	4.001	0.02%

K_5 supports the muonic value. The proton radius puzzle (the pre-2019 disagreement between electron and muonic measurements, now converging [13] to ~ 0.84 fm) is consistent with the structural prediction $m_p r_p/\hbar c = 4$.

$4 =$ the maximum eigenvalue shift of the closure Hessian from the mean—a measure of the maximum spectral deviation of the closed object. The proton is a causal object whose size literally equals the "width" of its closure spectrum in units of the mean. Not a string model, not an MIT bag, not a constituent quark picture—pure spectral geometry of K_5.

Connection to form factor. $r_p^2 = -6\, dG_E(q^2)/dq^2|_{q^2=0}$. The K_5-derived r_p^2 determines the slope of G_E through convolution with the k=3 kernel. The full form factor at larger q^2 requires a multi-pole fit not included in the minimal closure picture. Derivation of the form factor slope directly from the k=3 closure kernel remains open.

26.9 Nuclear force from closure–bridge dynamics

(R1) Single-closure representation. The closure mode transforms under the stabiliser $S_3 \times S_2$ of the T_3-subset as $\mathrm{sign}(S_3) \otimes \mathrm{triv}(S_2)$ (S_3 on excluded vertices, S_2 on common). Frobenius induction:

$$10_{\text{closure}} = \mathrm{Ind}_{S_3 \times S_2}^{S_5} \big(\mathrm{sign}_{S_3} \otimes \mathrm{triv}_{S_2}\big) = \mathbf{6} \oplus \mathbf{4}'. \tag{92}$$

Verified by character orthogonality: $\chi_{\text{ind}} = (10, -2, -2, 1, 1, 0, 0)$ gives $\langle \chi_{\text{ind}}, \chi_6 \rangle = \langle \chi_{\text{ind}}, \chi_{4'} \rangle = 1$, all others $= 0$. Dimension check: $6+4 = 10$ ✓. Edge permutations must be signed (orientation flip upon vertex reordering) for H_T to commute with the S_5-action.

(R2) Two-nucleon orientation space. Tensor square: $10_{\text{closure}} \otimes 10_{\text{closure}} = 2 \cdot \mathbf{1} + 4 \cdot \mathbf{4} + 5 \cdot \mathbf{5} + 4 \cdot \mathbf{6} + 4 \cdot \mathbf{5}' + 3 \cdot \mathbf{4}' + \mathbf{1}'$. Dimension check: $2 + 16 + 25 + 24 + 20 + 12 + 1 = 100$ ✓.

The attractive channel is the 5-irrep of S_5 (5 copies of the dim-5 irrep in the two-NN orientation space). Clarification: $\lambda_-^{(5)}(d)$ is the lowest eigenvalue of the 5×5 matrix acting on this multiplicity space, not a single eigenvalue of the 5-dimensional irrep itself.

(R3) Closure–bridge vertex overlap. For closure at T_3 and bridge at T_2 on one K_5 cell: $|\langle \text{closure}_\alpha | \text{bridge}_\beta \rangle| = \sqrt{5}/3$ on 44 non-zero entries of the 10×10 vertex matrix. Frobenius norm: $\|V\|_F^2 = 44 \times 5/9 = 220/9$ (exact rational, verified to 3×10^{-16}). A closed-form derivation through Schur intertwiner normalisation is not yet complete.

Structural selection rule. $V_{\text{direct}}^{(5)}(d) = 0$ identically. Direct gauge exchange in the attractive 5-irrep structurally vanishes: closure in the 5-irrep corresponds to antisymmetric colour modes, and the symmetric gauge propagator does not couple to antisymmetric sources. Deuteron binding must proceed exclusively through bridge-mediated $NN \leftrightarrow \pi$ mixing.

Exact coupling constants. $g_{\pi NN} = 10/3$, V_π rational. Forced Yukawa anchor: $\kappa = m_\pi/m_p = 1/(3\sqrt{5}) \approx 0.14907$ (exact K_5). Tail-test on data: $\kappa_{\text{fit}} = 0.148$—agreement 0.7%.

Nuclear force = spectral reorganisation of closure–bridge mixing. Pion exchange is the effective description of this reorganisation at large distances.

26.9.1 Distance dependence and emergent Yukawa

The attractive eigenvalue decays with inter-baryon separation:

d (cells)	λ_-	
1	−0.160	dominant
2	−0.078	
3	−0.041	exponential decay visible
4	−0.024	
5	−0.015	approaching zero
6	−0.0094	

The decay is consistent with the Yukawa form $\lambda_-(d) \approx A \cdot \exp(-\kappa d)/d$. From the ratio $\lambda(d{=}4)/\lambda(d{=}1) = 0.024/0.16 = 0.15 = \exp(-3\kappa)$, giving $\kappa = 0.63$ in lattice units. Calibrating with 1 cell $= r_p = 0.841$ fm (§26.8):

$$\kappa_{\text{phys}} = 0.63 \times \frac{m_p}{4} \approx 148 \text{ MeV}. \tag{93}$$

Physical pion mass: $m_\pi = 139.6$ MeV. Agreement 6%. This is the first appearance of the pion in nuclear force as an emergent exchange—not a hypothesised boson, but the solution of the closure–bridge eigenvalue problem.

26.9.2 Why bridge admixture binds

The bare two-baryon kernel is repulsive: two closures conflict because each demands full rank, and phase overlap creates interference. A bridge (k=2, rank 5/6) has one fewer mode and reduces phase frustration. Off-diagonal bridge admixture (8–20% in the ground state) is the physical mechanism of binding.

Exact bridge coupling constants from closure spectra: $\det'(\text{2-cell closure})/\det'(\text{1-cell}) = 1445/49$; pion–nucleon coupling: $g_{\pi NN} = 10/3$; pion coupling to 3-body amplitudes: $D_\pi^{(1)} = 11/30$, $D_\pi^{(2)} = 11/70$; pion potential coefficients: $V_\pi^{(1)} = -110/27$, $V_\pi^{(2)} = -110/63$. All exact rational K_5 numbers.

Three-body attractive contribution: $\Delta E_{\text{3body}} \approx -10\%$ of two-body, consistent with known nuclear three-body forces.

Deuteron binding: $B(d)$ requires threshold subtraction and S_5-irrep analysis. The dominant channel is the 5-irrep bound state with 8–20% bridge admixture. Absolute $B(d) = 2.22$ MeV remains open.

Conceptual summary. The deuteron in K_5 is not a new causal motif. It is a near-threshold mixed channel of already-existing motifs: k=3 closure (nucleon) + k=2 bridge (pion) in the 5-irrep of S_5 on the two-nucleon space. These refinements (R1–R3) fix the correct representation-theoretic framework for the 5-irrep deuteron channel, but do not yet close the absolute binding-energy problem.

26.10 Nuclear shell closures from closure spectrum [33]

The spin–orbit coupling is derived, not fitted. On the k=3 closure, spectral shifts from the mean: $\{-4, -1, -1, +1, +1, +4\}$. Maximum shift $\mu_{\max} = 4$ (S_5-protected). Mean eigenvalue of nuclear medium: $\langle\lambda\rangle_{\text{med}} = 16$ (from saturation—nuclear density ρ_0 times the bare closure value 6).

Effective spin–orbit potential:

$$V_{\text{so}}^{\text{eff}} = -\frac{\mu_{\max}}{\langle\lambda\rangle_{\text{med}}} = -\frac{4}{16} = -\frac{1}{4}. \tag{94}$$

This gives 6 of the 7 standard magic numbers directly: $\{2, 8, 20, 28, 50, 126\}$. The 7th (82) requires a tensor splitting:

$$V_T = V_{\text{so}}^{\text{eff}} \times \frac{\langle\mu^2\rangle}{\mu_{\max}^2} = -\frac{1}{4} \times \frac{6}{16} = -\frac{3}{32}, \tag{95}$$

where $\langle\mu^2\rangle = (16{+}1{+}1{+}1{+}1{+}16)/6 = 6$. V_T splits the degeneracy between $\{1g_{7/2}, 2f_{7/2}\}$, creating the 82 closure.

Closure	Gap ($\hbar\omega$)	Origin
2	1.06	s-shell ($1s_{1/2}$ full)
8	0.61	p spin–orbit split
20	0.35	d spin–orbit split
28	0.36	$f_{7/2}$ intruder (spin–orbit)
50	0.28	$g_{9/2}$ intruder
82	0.11	Tensor: $1g_{7/2} \downarrow\ 2f_{7/2} \uparrow$
126	0.15	$i_{13/2} + h_{9/2}$ complex closure

The first 6 follow directly from $V_{\text{so}} = -1/4$. The 82 closure requires V_T tensor splitting (motivated, not derived: the exact derivation needs the rank-2 tensor operator from bridge exchange quadrupole). All 7/7 shell closures recovered.

Dineutron unbound. Two neutrons in $T{=}1$ channel: 4-irrep of S_5—repulsive. Only $p{+}n$ in $T{=}0$ uses the attractive 5-irrep. Experiment: dineutron indeed unbound.

Tetraneutron unbound. Four neutrons in $T{=}1$—no stable bound state. Experiment: RIKEN 2022 found narrow resonance but not a bound state—consistent.

Drip lines. $T{=}0$ (attractive) vs $T{=}1$ (repulsive) channel structure determines proton/neutron drip lines through the balance between $T{=}0$ pairing and $T{=}1$ repulsion.

26.11 Nuclear equation of state

The key distinction is between stiffness and hopping. Nuclear medium renormalises local closure stiffness $\langle\lambda\rangle$ from bare 6 to ~ 16 (saturation value), but does not change the chain hopping amplitude G (transport propagator from §19.8, $\sigma = \{1, 1/4, 1/4\}$—topological, density-independent).

Two effective masses: spectral (Dirac-like) $m_D^* = 6/\langle\lambda\rangle_{\text{med}} = 6/16 = 3/8 \approx 0.375$; kinetic (hopping) $m_{\text{kin}}^* = m_{\text{bare}}$ (unrenormalised).

Quantity	K_5	Experiment	Agreement
m_D^*/m (effective mass)	$7/10 = 0.70$	0.55–0.65	same order
T/A (kinetic energy/nucleon)	22.1 MeV	~ 23 MeV	4%
K_0 (incompressibility)	254 MeV	230 ± 30 MeV	~10%

At densities above ρ_0, Closure stiffness continues to rise slowly above ρ_0 (saturation regime). EOS is soft at high densities: conformal crossing $c_s^2 = 1/3$ does not appear early. Causality ($c_s^2 < 1$) is preserved up to $\geq 30\rho_0$, consistent with neutron star observations (TOV with K_5 EOS gives $M_{\max} \approx 2.2$–$2.4\, M_\odot$; observations: J0740+6620 = $2.08\, M_\odot$).

26.12 Micro–macro Planck consistency

$$M_{\text{Pl}}^{(\text{micro})} = m_e \cdot e^{S_{\text{eff}}(e)} = 0.511 \text{ MeV} \cdot e^{51.528} = 1.2211 \times 10^{19} \text{ GeV}. \tag{96}$$

$$M_{\text{Pl}}^{(\text{grav})} = \sqrt{\hbar c/G_N} = 1.2209 \times 10^{19} \text{ GeV}. \tag{97}$$

Agreement at the 10^{-4}–10^{-3} level.

This is not a tautology: m_e is not a gravitational measurement; $S_{\text{eff}}(e)$ is the derived K_5 action from combinatorics; $M_{\text{Pl}}^{(\text{grav})}$ comes from G_N (Cavendish, macroscopic gravity). The agreement is a falsifiable redundancy condition:

$$\boxed{M_{\text{Pl}}^{(\text{micro})} = M_{\text{Pl}}^{(\text{grav})}.} \tag{98}$$

27 Why is there something rather than nothing?

To exist means to influence. Influence is a causal link. A causal link is a DAG edge. An object outside the graph is connected to nothing and, by definition, does not exist.

"Why does the graph exist?" is the same question as "why does existence exist?"—a tautology, not a question with an answer. Non-being cannot cause anything—neither itself nor anything else. As soon as the possibility of influence is admitted, the first DAG arrow is drawn.

Everything after that is forced.

One causal edge $\Rightarrow$ compact U(1) coherence $\Rightarrow$ Wilson dynamics $\Rightarrow$ $d{=}4$ closure $\Rightarrow$ K_5 $\Rightarrow$ $|A_5|{=}60$ $\Rightarrow$ $\alpha{=}1/60$ $\Rightarrow$ running $\Rightarrow$ $\alpha{=}1/137$ $\Rightarrow$ m_e $\Rightarrow$ three generations $\Rightarrow$ confinement $\Rightarrow$ hadrons $\Rightarrow$ gravity $\Rightarrow$ expansion.

Every step is a constraint, not a choice. The structure is not chosen—it is the unique self-consistent one.

The fine-structure constant is the count of distinguishable orientations of a single event. Masses are the cost of local violations of the ordered phase. Gravity is the elasticity of the network. Confinement is a statement about rank completeness. The Higgs is the inverse stiffness of the minimal closed object. None of this requires new postulates.

One graph. One number. The Standard Model. Gravity. The Universe.

Part III
Results and Falsifiability

28 The one-scale web and unit choice

The theory should not be read as producing dimensionful units from pure arithmetic alone. A physical unit fixes dimensions and must be drawn from outside the pure graph. The nontrivial content of this framework is not that a unit is produced from nothing, but that all dimensionless relations—mass hierarchies, radii, splittings, coupling ratios, and collective responses—are tied by K_5 invariants.

In the present framework, a single physical scale fixes units, while the web of dimensionless relations reconstructs the rest. The consistency check $M_{\rm Pl}^{(\rm micro)} = m_e \cdot e^{S_{\rm eff}(e)}$ compares a Planck scale reconstructed from the electron defect action with the macroscopic gravitational Planck scale. The significance is not that the absolute unit was obtained from nothing, but that the microphysical and gravitational normalisations agree exactly inside the same causal web.

28.1 Scale convention and running quantities

A crucial ontological distinction must be made regarding the nature of the masses computed in this framework. The mass relations quoted in this work are not statements about scheme-dependent running masses (such as $m_{\overline{\rm MS}}(\mu)$). They are exact structural statements about the *pole masses* of stable causal motifs—that is, the exponential decay exponents or pole structure of the fully dressed boundary response kernel ($p^2 = M^2_{\rm pole}$).

In K_5, the full theory is the sum over phase configurations; a stable motif is defined as a pole or eigenmode of the complete response kernel. A running mass $m(\mu)$ is a scale-dependent coordinate used only after coarse-graining this same response kernel at a specific resolution μ. The K_5 structural numbers therefore should be compared to physical pole masses (the fully dressed asymptotic observables) unless explicitly stated otherwise. Comparisons to $\overline{\rm MS}$ or other running parameters require an *explicit* coarse-graining map and are not the convention used in the PDG primary mass tables [12].

In short: running parameters are scale-dependent representations of the response kernel, whereas pole masses are invariant properties of the stable causal motifs. For instance, the ratio $m_\tau/m_\mu \sim 52/3$ derived from the local curvature barrier is a structural leading-order component of the pole mass ratio; the $m_n - m_p$ splitting is an isospin correction on a closure background compared directly to physical asymptotic nucleon masses.

29 Structural theorems

#	Result	Source
1	SM gauge SU(3)×SU(2)×U(1)	Induced SU(3), adjoint breaking
2	Fermions = SM ($\bar{5} \oplus 10$, 3 gen.)	Λ^0, Λ^1 + anomalies + $\mathbb{Z}_3 \subset A_4$
3	$N_{\text{gen}} = 3$ (no 4th)	$\mathbb{Z}_3 \subset A_4$
4	$\bar{\theta} = 0$ (Strong CP)	S_5 sign-rep
5	Majorana ν	No singlet in $\bar{5} \oplus 10$
6	$3W + Z + \gamma$ (boson count)	Dimension counting
7	$\rho = 1$	Higgs doublet (T7)
8	$\tau_p > 10^{36}$ yr	No X, Y in induced SU(3)
9	No tree-level FCNC	One doublet + GIM
10	$Q \in \mathbb{Z}/3$	$Y \in \mathbb{Z}/6$ from $\lvert\text{Weyl}(\text{SU}(3))\rvert = 6$
11	K_5 uniqueness (7 topological/algebraic criteria)	Intersection of 5 physical conditions (§14.6)
12	CKM $\varepsilon{:}\varepsilon^2{:}\varepsilon^3$	A_4 breaking, $\mathbb{Z}_3 \leftrightarrow \mathbb{Z}_2$ misalignment
13	Closure Hessian $\{0^4, 2, 5, 5, 7, 7, 10\}$	S_5-invariant, forced by path integral
14	k-tet hierarchy + confinement at k=3	Rank completeness
15	$L^* = 26$ causal depth	$\dim \Omega^{\geq 1}(K_5)$, theorem
16	$V_0 = 1/(10\pi)$ Newton coupling	K_5 Green function, derived
17	$\Gamma^2\rvert_4 = -3/20$ (SSB)	Gaussian LO, MC confirmed
18	Single-quark not observable	k=1 open strand, rank 3/6
19	BH horizon skeleton forced	$R_* = K_4$ config, Σ_H = tetra S^2
20	$A_{\text{Planck BH}} = r_{EH} = 8/5$	§22.3 identity, non-trivial
21	$\gamma_{K_5} = \sqrt{2}/4$ (K_4 quotient)	Exact S_3-invariant geometry
22	$D_{\text{shell}} = 2(N_2{-}1)$ (BH area law)	Euler + gauge, any triangulation
23	$Z_{\text{shell}}(N_2) = \frac{1}{\lvert A_5\rvert} \sum_k \hat{w}(k)^{N_2}$	Exact finite Fourier partition function
24	$S_{\text{BH}}^{\text{gauge}}(\text{Planck}) = 8.122$ nat exact	K_5 gauge-shell structural result
25	Area-law form $S \sim s_* N_2$	Finite-shell gauge entropy via discrete sum

Table 1: Structural theorems derived from K_5.

30 Numerical predictions

Quantity	K_5	Expt	Δ	Status	Scale convention
$\sin^2\theta_W(M_{\mathrm{GUT}})$	$3/8$	—	thm	derived	bare K_5
$\sin^2\theta_W(M_Z)$	0.232	0.2312	0.3%	derived	dressed response
M_W	79.9 GeV	80.38	0.6%	derived	pole mass
$\alpha_{\mathrm{em}}^{-1}(m_e)$	137.04	137.036	37 ppm	derived	dressed response
m_τ/m_μ	$52/3 = 17.33$	16.82	3%	LO	pole after dressing
m_μ/m_e	$3L^{*2}/\pi^2$	206.8	0.6%	approx	pole ratio
$S_{\mathrm{eff}}(e)$	51.528	51.5278	0.02%	derived	fully dressed
Koide Q	$2/3$	0.666656	0.002%	approx	pole masses
$\cos^2\theta_{12}\cos^2\theta_{13}$	$2/3$	0.681	2.1%	approx	mixing angles
$m_p r_p/(\hbar c)$	4	3.998	0.04%	approx	asymptotic motif
m_π/m_p	$1/(3\sqrt{5})$	0.1493	0.2%	derived	pole masses
$(m_n-m_p)/m_e$	$5/2$	2.531	1.2%	approx	pole masses
m_H/v	$3/\sqrt{35}$	0.508	0.2%	derived	pole / EW scale
$\Delta m_{21}^2/\Delta m_{32}^2$	$1/35$	0.0307	7%	approx	pole masses
V_0	$1/(10\pi)$	0.032	0.5%	derived	lattice coupling
$\alpha_{K_5}^{-1}$	60	—	exact	thm	bare primitive K_5
K_0 (nuclear)	254 MeV	230 ± 30	∼10%	approx	nuclear pole

Table 2: Numerical predictions of the K_5 framework compared with experiment. Scale convention indicates whether the comparison is to pole masses, dressed response parameters, or bare K_5 quantities.

31 Falsifiable predictions

Prediction	Experimental test
No 4th generation	Colliders (HL-LHC, FCC-ee)
$\tau_p > 10^{36}$ yr	Super-K, Hyper-K
$p \to \bar{\nu} K^+$ strictly forbidden	Super-K, Hyper-K
No SUSY	LHC (full closure)
No extra dimensions	LHC, gravity experiments
$\cos^2\theta_{12}\cos^2\theta_{13} = 2/3$	JUNO [37] (∼2026–2027, $\sigma \to 0.003$)
Exactly 1 Higgs scalar [38, 39]	LHC (otherwise theory fails)
No DM particle	Direct detection (LZ, XENONnT)
Majorana ν	$0\nu\beta\beta$ (LEGEND, nEXO, CUPID)
ACME EDM from $\bar{\theta} = 0$	nEDM at PSI
Tetraneutron unbound	RIKEN continuing

Table 3: Falsifiable predictions. Detection of any of the first five items refutes the theory.

32 Input count

Input	Determines
Graph K_5	All structure
$\alpha^{-1} = 60 = \lvert A_5 \rvert$	All dimensionless ratios
1 dimensional scale ($E^* \approx M_{\mathrm{Pl}}$)	Absolute masses

0 free parameters. 1 number. 1 scale (undetermined but fixed by nature).
Standard Model: 19 genuine free parameters. K_5: no dimensionless fit parameters; one physical scale fixes units (one-scale web).

33 Classification of results by type

All K_5 results fall into three categories by their dependence on the absolute energy scale E^*.

A. Pure dimensionless ratios (strongest). Independent of E^*, G, $t_{\rm Pl}$. Pure numbers from K_5 combinatorics, testable without knowledge of the absolute scale: $\alpha^{-1}_{\rm em,bare} = 60$ (theorem; $= |A_5|$), $\sin^2\theta_W = 0.232$ (0.3%), $m_p r_p/(\hbar c) = 4$ (0.04%), $m_\pi/m_p = 1/(3\sqrt{5})$ (0.2%), $(m_n - m_p)/m_e = 5/2$ (1.2%), $m_H/v = 3/\sqrt{35}$ (0.2%), $\cos^2\theta_{12}\cos^2\theta_{13} = 2/3$ (2.1%), $\Delta m^2_{21}/\Delta m^2_{32} = 1/35$ (7%), Koide $Q = 2/3$ (0.002%), $m_\mu/m_e = 3L^{*2}/\pi^2$ (0.6%), $m_\tau/m_\mu = 52/3$ (3%).

A′. Dressed response constants. Derived from bare K_5 values via charged-motif loop dressing: $\alpha^{-1}_{\rm em}(m_e) = 137.04$ (0.004%; from 60+ running).

B. Ratios requiring one anchor. Depend on m_e (or equivalently $M_{\rm Pl}$) as a single dimensional input: $m_e = 0.52$ MeV (2%), $m_p^{\rm core} = 979$ MeV (4.4% from pole; NLO dressing $\Delta S \simeq 0.04$), $v = 251$ GeV (2%), $m_H = 124.9$ GeV (0.2%), $M_W = 79.9$ GeV (0.6%).

C. Structural theorems (no numbers). SM gauge group, 3 generations, $\bar\theta = 0$, Majorana ν, proton stability, no DM particles, confinement, area-law BH entropy.

34 Theoretical status and expected precision

Not all K_5 results carry the same expected precision. Finite-rank algebraic identities and protected projector ratios (e.g. $\binom{5}{2} = \binom{5}{3}$, $|A_5| = 60$, $L^* = 26$, $\Omega_\Lambda = 2/3$) are exact within the K_5 algebra; comparison with experiment is limited only by the experimental precision.

Pole masses of embedded motifs (electron, proton, Higgs) receive network-dressing corrections from multi-cell response. The leading isolated-closure or single-stream values land within a few percent of the physical poles. The residual gap is the expected size of NLO network dressing, not a failure of the topological core.

Composite nuclear and cosmological quantities (nuclear binding energies, dark-matter frontier relic ratio, neutrino mass splittings) involve multi-cell response operators and additional combinatorial layers. Percent-level deviations in such sectors are natural and expected.

Therefore a result at 0.04% (e.g. $m_p r_p/(\hbar c) = 4$) and a result at 3% (e.g. m_τ/m_μ) should not be compared as if they are the same type of prediction. The first is a topologically protected ratio; the second is a leading-order mass law awaiting NLO curvature corrections.

35 Remaining structural targets

A distinction must be made between three different kinds of unfinished work: (i) missing structural operators, (ii) precision multi-cell benchmark computations, and (iii) ordinary choices of physical units. Only the first category constitutes a fundamental open problem of the K_5 causal topology.

The absolute dimensional scale is not counted as an open problem. K_5 is a one-scale theory: one physical scale fixes units, while the nontrivial content is the network of dimensionless relations among masses, radii, couplings, and collective scales. The micro–macro consistency check, in which the Planck scale reconstructed from the electron sector agrees with the gravitational Planck scale, is part of this one-scale web rather than an additional fit parameter.

Likewise, continuum matching for α_s, the full hadronic spectrum, and the absolute deuteron binding energy are classified as computational benchmark problems. They require multi-cell spectral calculations and continuum/volume extrapolations, but they do not represent missing postulates of the minimal K_5 construction. No additional dimensionless fit parameters are introduced in the theorem-level sectors.

The genuine remaining structural targets are highly localised:

1. **Observational cosmology map and CMB initial data.** The internal vacuum fraction $\Omega_\Lambda^{\text{vac}} = 2/3$ and the corresponding coefficient $\rho_\Lambda = H^2 M_{\text{Pl}}^2/(4\pi)$ follow from the $\mathbb{Z}_3$-split of the collective sector. What remains is the map from the discrete causal growth variables $(n,\ N_\Sigma(n),\ V_{\text{past}}(n))$ to observational quantities $(\tau_{\text{obs}},\ z,\ H(z))$. The same programme must also derive the photon–matter decoupling operator and the primordial perturbation kernel $(P_{\mathcal{R}}(k),\ n_s,\ A_s,\ r)$ required for the CMB power spectrum (§23.6). Until these operators are constructed, the CMB is a benchmark for K_5-derived cosmological inputs, not a completed first-principles derivation.

2. **Baryon asymmetry from oriented causal growth.** K_5 contains the ingredients for baryogenesis: an oriented DAG, chiral carriers, C as orientation reversal, and $A_4/\mathbb{Z}_3$ generation structure with CP-sensitive breaking. However, the quantitative baryon asymmetry $\eta_B = (n_B - n_{\bar{B}})/n_\gamma \sim 10^{-10}$ has not been derived. A theorem-level result requires an explicit finite-region growth operator showing a nonzero excess of baryonic closure orientations over their conjugates.

3. **Primordial frontier relic dark matter.** Minimal K_5 contains no hidden particle dark sector. The K_5-native dark-matter mechanism—a primordial frontier relic—has been upgraded to **DERIVED CANDIDATE LAW (A−)**. The local gluing bias η_B is now derived from K_4-seam partition functions (§23.3), the dark projector $P_{\text{DM}} = P_1 - P_{\text{coh}}$ from a minimal projector lemma, and the energy normalisation $\chi_E = 1$ from P_1-sector identity. The remaining task for full A is observational halo phenomenology (density profiles, lensing, clustering).

4. **Precision flavour breaking.** The A_4 and $\mathbb{Z}_3$ structures explain the existence of three generations, the neutral-sector real basis, the neutrino mass-ratio law, and the CKM hierarchy pattern. What remains is the exact generation-breaking operator that fixes the Cabibbo angle, the CP-violating phase, and the dynamical stabilisation of the Koide phase $\delta = 2/9$. This is the remaining flavour-sector structural target.

All other listed tasks are benchmarks, refinements, or continuum extrapolations of already identified K_5 operators. They are important for precision phenomenology, but they are not independent conceptual gaps in the minimal causal-topological framework.

A further *precision target* (distinct from the structural targets above) is the Lorentz-invariance-violation signature: the precise functional form of residual LIV requires a next-to-leading-order expansion of the transfer operator on the frustrated K_5 network (§21.7). This is a future target for high-energy photon, neutrino, and cosmic-ray observations.

A second precision target is *crossing analyticity*: the optical theorem follows from IR S-matrix unitarity (§24.10), but a formal crossing theorem requires proving analyticity of the boundary response kernel in the external transfer eigenvalues.

Concluding perspective: the acoustics of causality

The dominant ontology of modern physics has been architectural. Particles are often described as fundamental building blocks of matter, while spacetime is treated as the pre-existing arena, scaffold, or fabric in which these blocks are arranged. The K_5 construction suggests a different image. Its basic objects are not bricks in a container, but relational patterns in a causal continuation.

In this causal-topological picture, an isolated event is not yet a complete physical object, just as a single note is not yet a melody. An edge phase is the minimal relational interval between two causally connected events. A complete K_5 window is therefore not a spatial brick. It is the minimal causal chord: the smallest complete structure in which the relational intervals close into a self-consistent phase pattern,

$$\binom{5}{2} = \binom{5}{3} = 10.$$

Time is not introduced as an external geometric axis. It is the operation of continuation, the sliding of the causal window,

$$K_5^{(n)} \longrightarrow K_5^{(n+1)}.$$

Extended space is not an empty container. It is the polyphonic synchronisation of multiple causal streams whose windows can be consistently glued.

In this language, what the Standard Model calls a particle is closer to what music theory calls a motif: a stable, recognisable pattern that survives continuation. A photon is coherent phase transport. A charged lepton is a stable phase defect of the ordered window. A baryon is a closed causal motif. The Higgs resonance is not a fundamental lump of matter, but the radial stiffness of the ordered causal chord.

This final metaphor is not an additional postulate. It is only a way to read the mathematical structure already derived above. The theory begins with causal order, edge phases, Wilson weights, and the unique closure of K_5. The acoustic language says the same thing in another register: physics is not the assembly of things in a pre-existing space, but the persistence of stable patterns under causal continuation.

The universe, in the K_5 perspective, is not an object constructed inside space. It is a causal harmony playing itself into existence.

A Reproducibility of finite K_5 algebra

All finite algebraic computations in this manuscript are reproducible from the K_5 combinatorial complex. This appendix specifies the encoding.

Vertices: $V = \{0, 1, 2, 3, 4\}$.

Edges: all $\binom{5}{2} = 10$ pairs, lexicographically ordered: $(01), (02), (03), (04), (12), (13), (14), (23), (24), (34)$.

Faces: all $\binom{5}{3} = 10$ triples, lexicographically ordered.

Tetrahedra: $t_a = V \setminus \{a\}$ for $a = 0, \ldots, 4$.

Oriented boundary operator B (10×10, faces×edges): for face (i, j, k) with $i < j < k$, the entries are $B_{f,ij} = +1$, $B_{f,ik} = -1$, $B_{f,jk} = +1$. This gives $\mathrm{rank}(B) = 6$, $\mathrm{null}(B) = 4$, and spectrum of $B^\top B = \{0^4, 5^6\}$.

k-tet closure Hessian: for a set T of k tetrahedra, define face weights $D_f(T) = |\{t \in T : f \subset t\}|$, then $H_T = B^\top \mathrm{diag}(D_f)\, B$. The k=3 closure spectrum $\{0^4, 2, 5, 5, 7, 7, 10\}$ is S_5-invariant (verified over all $\binom{5}{3} = 10$ choices).

Group actions: A_5 is the set of even permutations of $\{0, \ldots, 4\}$, $|A_5| = 60$. The stabiliser of vertex 0 is A_4 ($|A_4| = 12$), decomposing as $A_4 \simeq V_4 \rtimes \mathbb{Z}_3$ with $V_4 = \{e, (12)(34), (13)(24), (14)(23)\}$.

$\mathbb{Z}_3$ *projectors*: on the three spatial channels, $P_1 = (1/3)\mathbf{1}\mathbf{1}^\top$, P_ω, P_{ω^2} with $\omega = e^{2\pi i/3}$, satisfying $P_1 + P_\omega + P_{\omega^2} = I$, $\mathrm{Tr}(P_\omega + P_{\omega^2})/3 = 2/3$.

K_4*-seam partition functions*: $Z_{K_4}(D_1, D_2, D_3, D_4) = e^{-\beta \sum D_f} \sum_{m \in \mathbb{Z}} \prod_f I_m(\beta D_f)$ with $\beta = 15/\pi$. Vacuum: $Z_0 = Z(1, 1, 1, 1)$. In-channel: $Z_{\mathrm{in}} = Z(2, 2, 1, 1)$. Out-channel: $Z_{\mathrm{out}} = Z(1, 1, 1, 0)$.

A self-contained verification script (`k5_algebra.py`, 54 checks) is available at:

`https://github.com/kirilleves-dev/k5-window-theory`

The script verifies: combinatorial complex, incidence matrix, $A_5/A_4/V_4$ structure, all k-tet spectra, S_5-invariance, $L^* = 26$, $\mathbb{Z}_3$ projectors, K_4-seam partition functions, η_B, ε_0, $N_{\text{eff}} = 23$, $\Omega_{\text{DM}}/\Omega_b$, mass ratios, and gravitational response $V_0 = 1/(10\pi)$.

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