

# The Ultimate Guide

Physics from Causal Generation

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## Abstract

We show that a generative causal process equipped with compact edge phase transport has a layered consequence: the primitive  $U(1)$  phase first contains a pure arithmetic cover branch, while placing that same phase on a directed causal network with Wilson curvature admits a unique minimal complete simplicial causal-gauge cell at the Hodge–Bianchi resonance dimension  $d = 4$ . In that dimension curvature and response are both middle-degree two-forms; the Bianchi and response equations live in the same equation-degree;  $F \wedge F$  is a top form; and the curvature sector admits a self-dual/anti-self-dual split. The minimal complete simplicial realisation of this  $d = 4$  closure is the 4-simplex, whose one-skeleton is the complete graph  $K_5$ . The familiar equality  $\binom{5}{2} = \binom{5}{3} = 10$  is therefore the simplicial ledger shadow of this middle-degree closure, not the primary selection principle.

$K_5$  is not a spacetime brick; it is a sliding causal window. Time is continuation; space is synchronisation. Particles are stable causal motifs: charged leptons are phase defects, baryons are  $k=3$  closures, pions are  $k=2$  bridges, photons are coherent transport of  $U(1)$  phase non-equivalence, the Higgs is the radial stiffness of causal ordering, and the  $k=4$  overclosure is the minimal local seed of a future-sealed black-hole cluster.

From this Hodge-closed causal cell we construct a one-scale web of theorem-level, derived-law, and candidate results, with no additional dimensionless fit parameters in the theorem-level sectors: the SM gauge group  $SU(3) \times SU(2) \times U(1)$ ; the minimal fermion carrier spectrum  $\bar{\mathbf{5}} \oplus \mathbf{10}$  (15 Weyl states, no sterile singlet) in three generations; the theorem-level bare orientation coupling  $\alpha_{\text{bare}}^{-1} = |A_5| = 60$ ; the electromagnetic conductor weight  $W_{\text{EM}} = \varphi(60)/2 = 8$  for charged channels, with the observed low-energy  $\alpha_{\text{em}}^{-1} \simeq 137$  obtained as an A–/LO dressed-response structure after lepton dressing and open-channel completion;  $\sin^2 \theta_W = 0.232$ ; the causal depth  $L^* = 26$ ; the Higgs ratio  $m_H/v = 3/\sqrt{35}$  (124.9 GeV, 0.2%); the proton radius law  $m_p r_p / (\hbar c) = 4$  (0.04%);  $m_\pi/m_p = 1/(3\sqrt{5})$  (0.2%); the neutrino law  $\Delta m_{21}^2/\Delta m_{32}^2 = 1/35$ ; the gravitational response  $V_0 = 1/(10\pi)$ ; the absolute scale bridge  $M_{\text{Pl}} = m_e \exp(51 + 19/36)$  (about  $10^{-4}$  agreement, A–/near-theorem, with denominator  $\text{Tr}'(d_2^\top d_2) = 30$ ); the operator-level vacuum split  $\Omega_\Lambda = 2/3$ ; and the dark-matter frontier relic candidate ratio  $\Omega_{\text{DM}}/\Omega_b \simeq 5.48$  (2.2% relative to the Planck 2018 ratio, with the number closed and halo phenomenology open).

Scattering is a boundary-to-boundary response kernel on the thermodynamically selected causal skeleton; unitarity is reconstructed in the infrared stable-motif sector; Lorentz invariance emerges as the kinematics of the surviving gapless response sector. Throughout the construction we use generated completeness as a working discipline: a quotient or readout may lose information, but it may not add an ungenerated degree of freedom. The companion arithmetic note treats ordinary arithmetic as the pure cover-shadow of the same compact phase and records RH as the corresponding cover-shadow completeness theorem; RH is not used by any finite physical derivation.

All claims are classified as theorem, derived law, candidate, benchmark, or open structural target.

**How to read this manuscript.** The text is intentionally bold, but it is not meant to blur what is proven and what remains open. A claim labelled **THEOREM** is an exact consequence of the  $K_5$  structure or its algebra. A **DERIVED LAW** is obtained from already established  $K_5$  objects and normalisations. A **CANDIDATE** is a mechanism with a causal chain and numerical support but with at least one operator-level step not yet closed. An **OPEN** problem is explicitly

not presented as solved. Throughout the manuscript we also impose a no-horizontal-import rule: a higher layer may inherit structures generated by lower layers, but it may not silently import an external analytic, spectral, or statistical object and call it derived without first constructing the corresponding  $K_5$ -object.

This first part is not a popular replacement for the technical text. It is a causal reading guide. Each radical claim is stated in ordinary language and then linked back to the same chain:

causal order  $\rightarrow$  edge phase  $\rightarrow K_5$  window  $\rightarrow$  gluing  $\rightarrow$  stable motif  $\rightarrow$  observed effect.

**How to read a radical claim.** A radical claim in this manuscript is never meant to stand alone as philosophy; it is to be read as a causal chain. For example, the statement that the Higgs resonance is “not a fundamental matter particle” is not a rhetorical statement about names. It means that the object identified experimentally as a 125 GeV scalar is represented mathematically in this theory as the radial ordering mode of the causal window ( $A_5 \rightarrow A_4$ ), whereas charged fermion masses are computed separately from topological defect costs. The claim is ontological, not observational: the resonance is real, but the primitive explanation is different.

The same discipline applies to Bell correlations, quarks, virtual particles, and dark matter. The manuscript does not deny the experiments. It changes the object to which the experimental language is attached. A useful translation rule: *observable phenomenon*  $\rightarrow$  *usual ontology of that phenomenon*. The theory keeps the former and re-derives the latter from causal topology.

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## Part I

# The Causal Picture

## 1 Starting before space and particles

Physics often begins with space, time, particles, and fields. This framework begins earlier. It starts from generated occurrences and their ancestry order: a new occurrence can only arise from an already existing boundary. This birth order induces a directed acyclic graph. At this point there is no background space, no particle inventory, and no force law. There are only generated occurrences and ancestry relations.

If two events are causally connected in the physical sense, they cannot be identical. The connection must carry a readable distinction. Here a causal edge is not an empty arrow in a bare partial order; it is the minimal physically distinguishable influence that can be transported, composed, compared with refinements, and responded to. The minimal continuous compact distinction on such an edge is a phase. A single phase on a single edge is not itself an observable; physical content appears when phases fail or succeed to agree around closed comparisons. This is the origin of gauge content in the theory.

**Layered generation and no horizontal imports.** The primitive phase does not generate only one branch of structure. It generates a layered ladder, and each higher layer is allowed to use only objects already produced below it:

$\mathcal{L}_0$  : U(1) phase, finite covers,  $\zeta$ , and finite twists,

$\mathcal{L}_1$  : U(1) placed on a causal DAG with Wilson curvature,

$\mathcal{L}_2$  : the minimal closed  $K_5$  window and its algebra,

$\mathcal{L}_3$  : sliding chains, seams, transfer, screening, and local dynamics,

$\mathcal{L}_4$  : networks, continuum collective response, gravity, cosmology, and black holes.

This is not a stylistic organisation. It is a rule of derivation. A result is called derived only when every object in it has been generated by a lower layer. If an external theorem or analytic structure is useful but its hypotheses have not yet been realised inside the  $K_5$  ladder, the result remains a candidate or an open bridge. This rule is what prevents, for example, importing a Hilbert–Pólya, Lee–Yang, de Branges, or Herglotz framework as a proof unless the corresponding  $K_5$  object has first been constructed.

**Generated completeness.** The same discipline has an ontological form: a generated quotient, readout, closure sector, or continuation may lose information, but it may not add a phantom degree of freedom. In short,

loss is allowed; phantom addition is not.

Equivalently, what exists in  $K_5$  is what is generated by an admissible causal process; there is no independent store of ungenerated hidden objects behind the generated record.

**Theorem 1.1** (Generated completeness principle). *Let  $f : A \rightarrow B$  be an admissible  $K_5$  map (projection/forgetting map, contextual readout, closure-sector reduction, or generated continuation). The output is defined as the generated image,*

$$B = \text{Im } f.$$

*Therefore every admissible output element, channel, or observable degree of freedom has a generating preimage or a generated operation. Equivalently,  $K_5$  maps may forget information, but they cannot create phantom degrees of freedom.*

If  $A$  is compact and  $f$  is continuous, then  $B$  is compact as a continuous image of a compact space. Every observable  $O_B$  on the output pulls back to an observable  $O_B \circ f$  on the input. The converse need not hold: many input observables may be forgotten. This is the operational content of the theorem. The output may be a lossy shadow, but it cannot contain an additional hidden layer not generated by the source.

Local dynamics for these phases must be gauge-invariant, local, and positive. The natural local action is therefore Wilson-type:

$$S = \beta \sum_f (1 - \cos \Phi_f), \quad (1)$$

where  $\Phi_f$  is the phase mismatch around a minimal face. The deeper selection filter is not the raw equality of edge and face counts. After gauge and Bianchi reduction, complete simplices have matching reduced cochain ranks in every dimension. The actual filter is Hodge–Bianchi closure: only in  $d = 4$  do curvature and response live in the same middle degree,  $F, *F \in C^2$ , while  $F \wedge F$  is a top form and the 2-form sector admits a self-dual/anti-self-dual split. The minimal complete simplicial realisation of  $d = 4$  is the 4-simplex, whose one-skeleton is  $K_5$ . The visible equality

$$\binom{5}{2} = \binom{5}{3} = 10 \quad (2)$$

is the simplicial ledger shadow of that middle-degree closure, not the primary selection principle.

The  $K_5$  window is not chosen as a convenient lattice cell. It is the minimal complete causal window capable of realising Hodge–Bianchi curvature-response closure.

**Layer-0 arithmetic branch.** The same primitive compact phase also has a purely arithmetic consequence before any causal geometry is placed on it. The circle admits finite orientation-preserving covers

$$\phi_n : S^1 \rightarrow S^1, \quad \phi_n(z) = z^n,$$

and composition of covers is multiplication of degrees,

$$\phi_m \circ \phi_n = \phi_{mn}.$$

Thus the pure phase layer contains the cover monoid

$$\text{Cov}^+(S^1) \cong (\mathbb{N}^\times, \times),$$

whose indecomposable elements are the ordinary primes. In native cover coordinates

$$n = \prod_p p^{a_p} \quad \longleftrightarrow \quad (a_2, a_3, a_5, \dots),$$

ordinary arithmetic appears as the free commutative cover gas of the phase circle, while the edge phase itself remains the one-dimensional group  $U(1)$ . The existence of the entire cover cascade is forced by compactness:  $\pi_1(S^1) = \mathbb{Z}$  classifies finite-index cover degrees, whereas a noncompact line would have  $\pi_1(\mathbb{R}) = 0$  and no native cover arithmetic. The identity-twist cover character

$$\chi_{\text{triv}}(n) = 1$$

has response

$$L(s, \chi_{\text{triv}}) = \zeta(s),$$

which the arithmetic companion calls the arithmetic photon: the untwisted  $U(1)$  response before the phase is placed on a causal DAG. The same companion note also proves that the finite conductor shell selected by  $q_{K_5} = |A_5| = 60$  has the carrier dictionary

$$(\widehat{\mathbb{Z}/60\mathbb{Z}})^\times = 1_\gamma \oplus \bar{\mathbf{5}} \oplus \mathbf{10}.$$

It also proves the unique alternating-conductor identity

$$\varphi(|A_n|) = 2^{n-1} \iff n = 5 \quad (n \geq 3),$$

so  $K_5$  is the unique alternating window whose orientation conductor has a  $2^{|V|-1} = 16$  unit packet. The geometric branch places the U(1) phase on a directed causal network and then imposes Wilson, Hodge–Bianchi, and sliding-window closure, producing  $K_5$  physics. The two branches share the same primitive phase. In the companion note the Riemann hypothesis is treated as generated completeness of this arithmetic cover-shadow: an inner/all-pass factor would be phantom phase, not forgotten information from the causal source.

A compact causal observer therefore does not meet arithmetic as an external language imposed on the world. Such an observer is inside the generated system, reads it through lossy maps, and lives on compact phase data. The only native catalogue of pure phase distinctions available before graph-geometry is the cover catalogue of  $S^1$ . Its degrees multiply, its indecomposable elements are primes, and its identity-twist partition function is  $\zeta$ . Natural numbers, multiplication, primes, and the zeta function appear because they are the unique self-observation language of a compact causal phase, not because mathematics is added from outside.

The  $K_5$  window is also not a brick of pre-existing space. It is a sliding window of a causal stream. Five successive ticks form one window; the next tick shifts the window by one step. Time in this theory is therefore not first a coordinate. Time is the operation of continuation. Extended space appears later, when many such streams synchronise through compatible windows.

The choice of  $K_5$  is not merely a geometric convenience. The abstract symmetry code of the five-vertex window ( $A_5$ , 60 elements) is the same code that produces icosahedral shells in chemistry and biology. In an ordinary spatial system, where all directions are reversible, this code naturally forms closed finite structures. But when constrained by generative ancestry, the same code cannot close along the causal direction: temporal closure would require an occurrence to belong to its own ancestry, which is not generable by boundary extension. Thus the very principle that creates stable, finite motifs in space is forced to become continued growth in time. Expansion is the temporal expression of the same closure principle that produces matter.

## 2 What exists: causal motifs, not particle bricks

The theory does not first postulate particles. It classifies stable causal motifs of the  $K_5$  window. A motif may be a local defect, a bridge, a closure, a neutral carrier, a radial ordering mode, or a collective network mode. Ordinary particle language appears only after such motifs have been identified.

This changes the status of familiar objects. An electron is not the same kind of object as a proton. A proton is not three little balls. A pion is not a stable primitive object in the same sense as a proton. The Higgs resonance is real, but it is not a fundamental matter particle. A photon is not a light-ball moving through a pre-existing space. These are different motifs, with different stability conditions.

## 3 Photon as phase non-equivalence transport

In this framework a photon is not a small object flying through space. It is the coherent transport of U(1) phase non-equivalence between successive causal windows. The phase lives on causal edges; triangular holonomy measures the mismatch between a direct ancestry relation and its refined relation. When this mismatch is transported coherently through sliding  $K_5$  windows, the result is the photon.

*photon = coherent transport of phase non-equivalence through sliding  $K_5$  windows.*

Frequency is the rate of phase rotation per causal continuation step. Polarisation is the way that this surviving phase response is distributed across the spatial channels of the window. Nothing here requires a small object flying through an already-given space. Space is the synchronised large-scale organisation of the same windows. Technically, this becomes the massless pole of the surviving compact  $U(1)$  response branch in §19.9.

There is an exact Layer-0 analogue before the phase is placed on a graph. The identity twist of the cover monoid has response

$$L(s, \chi_{\text{triv}}) = \zeta(s).$$

Thus the physical photon and the arithmetic photon are not the same object, but they occupy the same categorical slot: the untwisted  $U(1)$  response. In the physical branch the protection is the massless pole of the coherent seam response; in the arithmetic branch the same protection is expressed as generated completeness of the scale-stress completed identity-twist response.

*Status: Causal interpretation. The technical gauge sector and edge-phase construction are part of the main body; the arithmetic photon, finite-twist packet, and generated-completeness form of RH are stated in the companion note.*

## 4 Bell correlations without magical transmission

Bell-type experiments are often presented as a puzzle about two distant particles. The particles separate. Later one chooses a measurement on the left and a measurement on the right. The outcomes are correlated more strongly than a classical local hidden-answer picture allows. It is then tempting to say that one particle somehow tells the other what happened.

The  $K_5$  view rejects the first sentence of that story. The primary object is not two independent particles. The primary object is one causal motif with a common phase structure and common gluing constraints. What is later described as two distant systems are two contextual readouts, or two distant boundaries, of that earlier motif.

The usual local hidden-variable factorisation assumes that, once some hidden state  $\lambda$  is given, the left and right responses are independent:

$$P(A, B|x, y) = \sum_{\lambda} \rho(\lambda) P(A|x, \lambda) P(B|y, \lambda). \quad (3)$$

In words: each side is supposed to carry a ready answer to every possible local question, and the joint result is built by multiplying left and right local answer tables.

This is not the  $K_5$  ontology. The hidden state is not a pair of independent little objects. It is one causal motif. A measurement setting is a choice of contextual boundary readout of that motif. Two incompatible readout contexts do not have to coexist as a pre-written table of facts. Therefore the compatibility of two local outcomes need not factor into a left answer multiplied by a right answer. A more faithful causal form is

$$P(A, B|x, y) = \sum_{\lambda} \rho(\lambda) C_{\lambda}(A, B; x, y), \quad (4)$$

where  $C_{\lambda}$  is a compatibility condition for two contextual readouts of one common motif, not a product of two independent tables.

There is no new superluminal causal arrow from left to right. The correlation is not transmitted at the moment of measurement. It is inherited from the earlier common motif and from the fact that the two later outcomes must be compatible readouts of that motif.

Bell correlations do not force magical transmission. They force the rejection of the particle-first assumption that two separated systems already carry complete independent answer tables.

Einstein was right to reject mysterious action at a distance. The  $K_5$  theory rejects it as well. But the classical alternative is also wrong: the world is not a set of independent little objects

equipped with all possible answers. The primary object is the causal motif, and measurement selects a contextual readout of it.

*Status: Causal-topological interpretation. The full derivation of the Born rule, correlations, and decoherence belongs to the technical quantum-sector sections.*

## 5 Higgs: a real resonance, not fundamental matter

The common slogan says that the Higgs gives mass to particles. In the Standard Model as a parameterisation this is useful, but it does not explain the fermion masses: the Yukawa constants are free inputs.

In  $K_5$  the fermion masses are not put in through Yukawa constants. They arise as causal costs of stable motifs: defects, chain screenings, and closures. The electron, muon and tau hierarchy is controlled by local curvature and causal-depth screening. The proton mass is controlled by a closure action. Thus the Higgs is not needed as the origin of fermion masses.

What, then, is the Higgs? The  $K_5$  window has an  $A_5$  symmetry. The causal ordering selects an  $A_4$  stabiliser. The four-dimensional representation decomposes as  $4 \rightarrow 1 \oplus 3$ . The three directions are Goldstone-like directions of the broken orientation. The remaining singlet is the radial stiffness of the ordered phase. The observed 125 GeV resonance is identified with this radial mode.

$$\text{Higgs} = \text{radial stiffness of the causal ordering } A_5 \rightarrow A_4.$$

The resonance is real. Its ontological status is different: it is not fundamental matter, but a collective ordering mode of the causal window.

*Status: Derived law for  $m_H/v$  in the technical body; interpretation as radial ordering mode is central to the  $K_5$  ontology.*

## 6 Quarks and baryons

A coloured object is not a complete stable causal motif. It is a channel of a larger motif. The proton is not three independent little constituents arranged in space. In  $K_5$  it is the minimal colour-neutral  $k=3$  closure. What ordinary language calls a quark is a coloured channel inside a closure description.

$$\text{quark} = \text{coloured channel of an unfinished closure motif.}$$

This is why the theory should not seek a free quark mass as if it were the mass of an isolated object. The stable physical object is the closure. The colour channels are meaningful inside the closure, not as standalone causal objects.

*Status: Closure-sector spectra and baryonic observables are technical results; the no-standalone-quark interpretation follows from the closure ontology.*

## 7 The new primitive $L^* = 26$

The number 26 is not a spatial length and not a number of cells in the universe. It is the full non-point relational content of one  $K_5$  window:

$$L^* = \dim \Omega^{\geq 1}(K_5) = 10 + 10 + 5 + 1 = 26. \quad (5)$$

The terms are edges, faces, tetrahedra, and the top form. Vertices are point events; they do not carry relational content by themselves.

$$L^* = 26 \text{ is a new kind of physical primitive: causal relational depth.}$$

It controls chain screening and appears in the charged-lepton hierarchy. It is not fitted. It is the number of non-point relations a  $K_5$  window can carry.

*Status: THEOREM in the technical body.*

## 8 Virtual particles

In perturbative quantum field language, a virtual particle is an internal line of a diagram. It is not observed as an incoming or outgoing particle and need not satisfy an on-shell mass relation. Even in that language it is not a particle in the same sense as a detected electron or photon.

In  $K_5$  the distinction becomes ontological. The fundamental object is a causal window with phases on edges. On a finite region the theory sums over admissible phase configurations:

$$Z = \int_{U(1)^E} \prod_e d\theta_e \exp(-\beta \sum_f (1 - \cos \Phi_f)). \quad (6)$$

There are no little internal objects flying between events inside this expression. There are phase configurations, gluing constraints, and response kernels.

A real observed particle is a stable causal motif: a defect, bridge, closure, carrier, radial ordering mode, or collective transport mode. A virtual particle does not pass this stability test. It is a contribution of unstable or off-saddle phase configurations to the response of the network.

For example, when two charged defects interact, the  $K_5$  description is not that a small virtual photon flies between them. The interaction is the correlated response of the  $U(1)$  phase sector between two charged motifs. In perturbative language this response can be drawn as an internal photon line. In  $K_5$  language it is a phase-response kernel.

*virtual photon = shorthand for a  $U(1)$  phase-response kernel, not a flying object.*

The same reading applies to loop effects. They are corrections from nonlinear and anharmonic parts of the phase action around an ordered configuration. Diagrams may be convenient, but they are not the ontology.

## 9 Dark energy and dark matter

Dark energy is not introduced as a new field. In the  $K_5$  collective sector the  $\mathbb{Z}_3$  decomposition gives a matter/vacuum split. The internal vacuum fraction is

$$\Omega_\Lambda^{\text{vac}} = \frac{2}{3}, \quad (7)$$

and the corresponding coefficient in

$$\rho_\Lambda = \frac{H_0^2 M_{\text{Pl}}^2}{4\pi} \quad (8)$$

is the same statement in standard cosmological normalisation.

Dark matter is different. The minimal theory contains no hidden particle dark sector. The only surviving  $K_5$  mechanism is a primordial frontier relic: an excess of nonluminous causal structure seeded by baryonic closures during early network growth and then frozen into the mature network. The leading ratio is now an  $A-$  derived candidate law: the frontier freeze-out operator gives  $\Omega_{\text{DM}}/\Omega_b \simeq 5.48$ . The number is closed at the  $K_5$  operator level; what remains open is astrophysical halo phenomenology—density profiles, lensing response, clustering, and survival through structure formation.

*Status: Dark energy: derived/operator-level in the technical body. Dark matter: candidate frontier-relic mechanism.*

## 10 What this theory changes

The theory does not merely add a field or a parameter. It changes the order of explanation.

- Space is not first; causal continuation is first.
- Particles are not first; stable causal motifs are first.
- The Higgs resonance is real, but it is not fundamental matter.
- Quarks are coloured channels of closure motifs, not standalone objects.
- Bell correlations do not require action at a distance; they reveal the failure of independent answer tables.
- Virtual particles are not objects of the world; they are terms in a response expansion.
- Dark energy is a collective vacuum-sector split, not an added field.
- Particle dark matter is absent in the minimal theory; the only current  $K_5$  route is a primordial frontier relic.
- The number  $L^* = 26$  is a new physical primitive: relational depth.
- Scattering cross sections are boundary-to-boundary response kernels of the phase network, not exchanges of virtual objects. The Rutherford, Mott, and Møller results are recovered structurally.

## 11 What would falsify the minimal theory

A radical theory should expose itself to clear failure modes. The minimal  $K_5$  version would be put under direct pressure by any of the following:

- a fourth Standard-Model-like generation;
- a stable fundamental particle dark-matter sector;
- proton decay through heavy  $X, Y$ -type gauge bosons;
- neutrinos forced to be Dirac particles by experiment;
- tree-level flavour-changing neutral currents not accounted for by the  $K_5$  hierarchy;
- a failure of the dimensionless closure laws, such as  $m_p r_p / (\hbar c) = 4$  or  $m_\pi / m_p = 1 / (3\sqrt{5})$ ;
- a failure of the radial-ordering relation  $m_H / v = 3 / \sqrt{35}$ .

The theory is therefore not protected by adjustable parameters. Its strength is also its risk: many observables are tied to the same causal window.

## 12 Relation to adjacent programmes

The present construction is adjacent to several research programmes that also begin from discrete or causal structures: causal sets [19, 20, 21], causal dynamical triangulations [22, 23], spin foams and loop quantum gravity [24, 25], Regge calculus [5], lattice gauge theory [1, 34], and deterministic/cellular-automaton approaches to quantum mechanics [26].

It differs from all of them in one structural point: the local building block is not a generic simplex, a generic graph-update rule, or a generic spin network. It is the unique self-contained Hodge-closed causal-gauge window  $K_5$ , with  $\binom{5}{2} = \binom{5}{3} = 10$  as its simplicial ledger signature, extended by sliding continuation. This uniqueness is the origin of the predictive power claimed in the present work: the entire particle spectrum, gauge structure, and cosmological parameters follow from one object, not from a class of models.

The closest relative is the causal set programme [19], which shares the starting point of causal order but does not impose compact edge phases, does not have a gauge structure, and does not select a unique local window. Lattice gauge theory [1] shares the Wilson action but starts from a regular lattice in a pre-existing spacetime, whereas  $K_5$  derives both the lattice and the spacetime from generative causal growth plus minimal compact edge distinguishability and local closure.

## 13 What is deliberately not claimed

A clean manuscript must be as precise about what it does not claim as about what it claims.

The minimal theory does not claim that every numerical observation has been derived from pure algebra. It does not claim that the Hubble tension is definitively solved. It does not claim that the dark-matter frontier relic is already a closed theorem. It does not claim that every nuclear binding energy is known from a closed  $K_5$  formula.

What it does claim is that many of the dimensionless structures normally treated as independent—mass hierarchies, couplings, collective responses, shell closures—are shadows of one causal window. The remaining gaps are localised open operator problems inside the  $K_5$  network, not hidden free parameters.

## Part II

# Technical Derivation

## 14 From generative growth to $K_5$

### 14.1 Generated occurrences and ancestry order

Suppose that anything happens at all. What happens is not yet a particle, a point of space, or a field value. It is a generated occurrence: a minimal token of distinguishability in a growing relational network. A new occurrence can only be generated from an already existing boundary. Thus every occurrence has an ancestry. The ancestry relation induces a directed acyclic graph: arrows point from earlier generated occurrences to later ones. A causal loop is not an additional forbidden configuration; it is simply not generable by boundary extension.

Neither space, nor particles, nor forces, nor quantum mechanics are assumed. Only generated occurrences and their ancestry order are used.

## 14.2 One-dimensional ancestry is empty

A linear chain

$$v_0 \prec v_1 \prec \cdots \prec v_n$$

with only nearest-neighbour ancestry links has  $n$  edges and  $n+1$  vertices. The vertex phase redundancy removes one degree of freedom per vertex modulo one global overall phase, hence  $n$  gauge degrees of freedom. Therefore the number of physical edge degrees of freedom is

$$n - n = 0.$$

A one-dimensional causal history is pure gauge. It carries no holonomies, no action, and no observable local physics. To possess even a single physical degree of freedom, the generated stream must develop transverse ancestry relations. Transverse relations create the first triangular refinements, and with them the first gauge-invariant observables: holonomies.

## 14.3 What a primitive edge can carry

Two ancestrally related occurrences are connected by an edge. The edge cannot be null: if it carries no distinction, the two occurrences are not distinct within the generated network.

The primitive edge datum must satisfy four requirements:

1. *Invertibility*: reversing the comparison inverts the datum,  $g(s \rightarrow t) = g(t \rightarrow s)^{-1}$ .
2. *Composition*: ancestry chains compose their edge data.
3. *Continuity*: arbitrarily small mismatches are possible.
4. *Minimality*: one primitive edge carries one continuous degree of freedom and no hidden internal index beyond source, target and orientation.

The datum must also be compact, otherwise the local phase measure is not finite. The unique compact, connected, one-dimensional Lie group is  $U(1)$ . Thus a primitive edge carries a phase

$$g_e = e^{i\theta_e}, \quad \theta_e \in S^1.$$

**Theorem 14.1** (Primitive edge rigidity). *Let the primitive edge datum be nontrivial, invertible, composable along ancestry chains, compact, connected, and one-dimensional. Then the edge group is isomorphic to  $U(1)$ .*

**Corollary 14.2.** *Non-abelian gauge structure ( $SU(3)$ ) arises not from edges but from cells—combinatorial objects with internal structure. The hierarchy is:  $U(1)$  on edges (rank 1),  $SU(3)$  on cells (rank 2, from  $10 \rightarrow 8$  via tracelessness).*

## 14.4 Triangular holonomy and the Wilson action

A triple

$$v_i \prec v_j \prec v_k$$

contains a direct ancestry relation  $v_i \prec v_k$  and its one-step refinement  $v_i \prec v_j \prec v_k$ . The holonomy

$$\Phi_{ijk} = \theta_{ij} + \theta_{jk} - \theta_{ik}$$

measures the phase mismatch between these two causal descriptions of the same boundary relation. This is not a spatial loop and not a backwards causal arrow. It is the minimal discrepancy between a direct relation and a refined relation.

The local action must be a periodic function of  $\Phi$ , because  $\theta \in S^1$ , and it must penalise large mismatches. The minimal leading periodic representative with a minimum at  $\Phi = 0$  is the Wilson term [1]:

$$S_W = \beta \sum_f (1 - \cos \Phi_f).$$

More general local periodic actions may contain higher harmonics  $(1 - \cos n\Phi)$ . These modify ultraviolet and next-to-leading corrections, but the Wilson term is the leading representative of the ordered infrared universality class [34].

Gauge transformation:  $\theta_k \rightarrow \theta_k + \lambda(\text{target}) - \lambda(\text{source})$ , where  $\lambda(v)$  is an arbitrary phase at vertex  $v$ . Observables are gauge-invariant combinations only (holonomies). Charge quantisation follows from compactness:  $\theta \in [0, 2\pi)$  lives on a circle, not a line, and this automatically forces electric charge into integer units.

## 14.5 From triangular refinements to the causal cell

Consider a local ancestry interval containing  $n$  generated occurrences. If the interval is treated as a saturated comparison ledger, then every pair of occurrences carries one effective pairwise ancestry-comparison slot, and every triple carries one elementary triangular refinement record comparing a direct relation with its one-step refinement through an intermediate occurrence. These comparison edges are not Hasse-cover birth links; they are effective transport slots in the saturated local ledger.

It is tempting to look at the raw equality

$$\binom{5}{2} = \binom{5}{3} = 10$$

as the selection principle for  $K_5$ . This equality is important, but it is not a gauge-reduced degree-of-freedom theorem. For a complete simplex on  $n$  vertices, the curvature map satisfies

$$\text{rank } d_1 = \binom{n}{2} - (n - 1) = \binom{n - 1}{2}.$$

Thus the gauge-reduced edge rank equals the independent triangular-curvature rank for every complete simplex  $K_n$ . Reduced cochain rank alone therefore does not select  $K_5$ .

The actual structural filter is dimensional closure of the curvature-response complex.

## 14.6 Hodge–Bianchi resonance

**Theorem 14.3** (Hodge–Bianchi resonance). *Let the local cell have dimension  $d$ , with edge connection*

$$A \in C^1,$$

*triangular Wilson curvature*

$$F = dA \in C^2,$$

*Bianchi identity*

$$dF = 0 \in C^3,$$

*and Hodge response*

$$*F \in C^{d-2}.$$

*Then the curvature-response complex closes on one middle-degree algebra if and only if  $d = 4$ .*

*Proof.* The curvature-response sector closes on itself precisely when the curvature  $F$  and the dual response  $*F$  have the same degree:

$$2 = d - 2.$$

This gives  $d = 4$ . The Bianchi equation has degree 3, while the response equation has degree  $d - 1$ :

$$dF \in C^3, \quad d * F = J \in C^{d-1}.$$

These two equation-degrees coincide precisely when

$$3 = d - 1,$$

again giving  $d = 4$ . At the same dimension,  $F \wedge F \in C^4$  is a top form, and the middle-degree two-forms admit the self-dual/anti-self-dual split

$$\Lambda^2 = \Lambda_+^2 \oplus \Lambda_-^2.$$

These are not independent dimensional coincidences; they are equivalent facets of the single middle-degree condition  $2 = d - 2$ .  $\square$

The minimal complete simplicial realisation of a  $d = 4$  geometry is the 4-simplex  $\Delta^4$ . Its one-skeleton is the complete graph  $K_5$ . Therefore  $K_5$  is selected not by naive reduced-rank counting, but as the minimal complete simplicial causal-gauge cell capable of realising Hodge–Bianchi resonance.

**Role of the ledger count.** The combinatorial equality

$$\binom{5}{2} = \binom{5}{3}$$

is the simplicial ledger shadow of the same  $d = 4$  resonance. For a  $d$ -simplex,

$$N_1 = \binom{d+1}{2}, \quad N_2 = \binom{d+1}{3},$$

and  $N_1 = N_2$  occurs exactly when  $d = 4$ . Equivalently, in a saturated local ancestry ledger with  $n$  occurrences,

$$N_{\text{pair}}(n) = \binom{n}{2}, \quad N_{\text{ref}}(n) = \binom{n}{3},$$

so

$$R(n) = \frac{N_{\text{ref}}(n)}{N_{\text{pair}}(n)} = \frac{n-2}{3}.$$

Thus  $R(n) < 1$  for  $n < 5$ ,  $R(5) = 1$ , and  $R(n) > 1$  for  $n > 5$ . This transition is a statement about the primitive ledger of pairwise comparison slots and triangular refinement records, not about reduced cochain rank.

Intervals with  $n < 5$  are valid substructures: triangles, bridges, and  $K_4$  seams. Their limitation is not a missing gauge-invariant curvature mode after reduction; it is that they are relation-heavy ledgers rather than complete Hodge-closed cells. Intervals with  $n > 5$  are not inconsistent; they are non-minimal as primitive ledgers. If a complete  $K_n$  ledger is enlarged by one occurrence before sliding, then

$$\Delta N_{\text{pair}} = n, \quad \Delta N_{\text{ref}} = \binom{n}{2},$$

and the marginal refinement load is

$$\frac{\Delta N_{\text{ref}}}{\Delta N_{\text{pair}}} = \frac{n-1}{2}.$$

Forcing  $K_5$  to become a primitive  $K_6$  therefore adds five pairwise slots but ten triangular refinement records. The balanced continuation is not a new primitive  $K_6$  cell, but a sliding  $K_5$  window through the shared tetrahedral seam:

$$K_5^{(t)} \cap K_5^{(t+1)} = K_4.$$

**Status of the  $K_5$  selection.** The emergence of  $K_5$  is not a consequence of an undecorated partial order viewed merely as the relation  $x \prec y$ . A bare order records ancestry, but not yet physical influence.

The present construction uses causality in the stronger physical sense: a causal relation is a distinguishable influence that can be transported, composed, compared with refinements, and assigned a local response. In this sense the structures below are not independent additions to causality, but the internal conditions for causality to be physically readable.

Compact edge phase is the minimal continuous compact alphabet of causal distinguishability. Triangular holonomy is the minimal comparison between a direct causal relation and its one-step refinement. The Wilson term is the minimal positive gauge-invariant local cost of such mismatch. Hodge–Bianchi resonance is the condition that curvature and causal response close in one middle-degree algebra.

Thus the selection chain is

$$\begin{aligned} \text{physical/readable causality} &\Rightarrow \text{compact edge distinguishability} \Rightarrow \text{triangular Wilson curvature} \\ &\Rightarrow \text{Hodge–Bianchi closure} \Rightarrow d = 4 \Rightarrow \Delta^4 \Rightarrow K_5. \end{aligned}$$

The equality

$$\binom{5}{2} = \binom{5}{3} = 10$$

is the simplicial ledger shadow of this causal-response closure, not a standalone reduced-rank counting theorem.

**Corollary 14.4** (Hodge closure). *In the Hodge–Bianchi resonance dimension  $d = 4$ , the following statements coincide:  $F$  and  $*F$  have the same degree;  $\Lambda^2$  admits a self-dual/anti-self-dual decomposition;  $F \wedge F$  is a top form; and the simplicial ledger satisfies  $\binom{d+1}{2} = \binom{d+1}{3}$ .*

**Theorem 14.5** (Completeness of the minimal causal-gauge cell). *Let  $\mathcal{C}$  be a finite local cell satisfying: (1) transporters live on saturated pairwise comparison slots; (2) the local curvature is triangular Wilson curvature; (3) the cell realises Hodge–Bianchi closure; (4) minimal complete simplicial realisation. Then the 1-skeleton of  $\mathcal{C}$  is the complete graph  $K_5$ .*

**Theorem 14.6** (Gauge consistency on  $K_5$ ). *If the curvature  $F_{ijk} = 0$  for all ten triangular comparisons of  $K_5$ , the configuration is pure gauge ( $\theta_{ij} = t_i - t_j$ ). Any non-trivial gauge-invariant edge-phase information in a  $K_5$  window appears as nonzero triangular holonomy.*

Verified:  $\text{rank}(D^\top D) = 6$ , null space = gauge space (dim 4).

## 14.7 Additional consistency checks

The Hodge–Bianchi resonance selects  $d = 4$ , and minimal complete simplicial realisation selects  $K_5$ . The following are downstream consistency checks, not independent derivations of  $K_5$ . They show that later representation, anomaly, and response requirements converge on the same Hodge-selected cell:

**Condition 1:**  $d = 4$ .  $\binom{n}{2} = \binom{n}{3}$  only for  $n = 5$ . Additionally:  $A_n$  has a pair of inequivalent 3-dimensional irreps (required for chirality) only for  $n = 5$ . The spinor-vector coincidence  $2^{d/2} = d$  holds for both  $d = 2$  and  $d = 4$ ; it becomes physically relevant here only when combined with the Hodge–Bianchi curvature-response closure, which selects  $d = 4$ .

**Condition 2: Cell gauge algebra.**  $A_5$  supplies the finite orientation/cell symmetry from which the  $SU(3)$  cell algebra is induced in the later representation layer;  $A_5$  is not itself  $SU(3)$ . Smaller windows do not contain the required cell-algebra structure, while larger windows are overconstrained composites rather than new primitives.

**Condition 3: Fermion spectrum.** Only  $K_5$  gives  $5 + 10 = 15$  dimensions of  $\bar{\mathbf{5}} \oplus \mathbf{10}$ , coinciding with the minimal SM left-handed Weyl carrier ledger without a sterile singlet. The  $SO(10)$ -like 16 would be  $\mathbf{1} \oplus \bar{\mathbf{5}} \oplus \mathbf{10}$ , but the singlet is absent here.  $K_4$  gives  $4 + 6$ ;  $K_6$  gives  $6 + 15$ —neither reproduces the minimal SM ledger.

**Condition 4: Anomaly cancellation.** Only with  $\bar{\mathbf{5}} \oplus 10$  ( $K_5$ ) does anomaly cancellation yield a unique solution—SM charges.

**Condition 5: Strong CP.**  $\text{Sym}^2(\text{face rep})$  for  $S_5$  contains exactly 1 copy of the sign representation  $\rightarrow$  a unique pseudoscalar  $Q$ , and  $\bar{\theta} = 0$  is protected.

Condition	$K_3$	$K_4$	$K_5$	$K_6$	$K_{7+}$
$d = 4$ (Hodge resonance)	✗	✗	✓	✗	✗
Chirality (Out = $\mathbb{Z}_2$ )	✗	✗	✓	✓	✗
Spinor-vector + Hodge closure	✗	✗	✓	✗	✗
$SU(3)$ gauge	✗	✗	✓	?	?
Fermions $\bar{\mathbf{5}} \oplus 10$	✗	✗	✓	✗	✗
SM uniqueness	✗	✗	✓	✗	✗
$\bar{\theta} = 0$	—	—	✓	?	—

The intersection is a single point. This is not the primary selection argument, but a convergence of downstream consistency checks on the Hodge-selected  $K_5$  cell.

## 14.8 Why the same $A_5$ code closes in space but grows in time

The abstract  $A_5$  code appears in many closed spatial structures: icosahedral molecular shells ( $C_{60}$  fullerenes, viral capsids, clathrin cages, boron clusters). This does not mean that those systems and the  $K_5$  causal window are the same object. The point is more precise: the same symmetry code behaves differently depending on whether it acts in reversible space or in an oriented causal graph.

In an ordinary spatial system the graph is effectively undirected. Rotations and rearrangements have inverses. The free-energy problem is solved within a group action, and symmetric closed orbits are allowed. This is why icosahedral  $A_5$  structures naturally form closed finite shells.

The  $K_5$  causal window is different. It lives first in a directed acyclic graph. The causal arrow does not supply a generation procedure for a future-to-past inverse move. Therefore the time evolution of the window is not a group action but an oriented continuation:

$$K_5^{(n)} \longrightarrow K_5^{(n+1)}.$$

Fixing a causal direction breaks the  $A_5$  symmetry of the unoriented window to the tetrahedral stabiliser  $A_4$ . Spatial substructures can still close: this is what happens in  $k=3$  baryonic closures

(§26.4) and in future-sealed horizon clusters (§22.4). But closure along the causal arrow would be a time loop and is not generable by boundary extension.

Thus  $K_5$  does not forbid closure. It distinguishes spatial closure, which is generable inside an already produced slice, from temporal closure, which has no generation procedure. The same closure principle that makes finite symmetric shells in space becomes continued growth when projected along the causal direction.

## 15 The Sliding Window

### 15.1 $K_5$ as a sliding window and 3+1

$K_5$  is not a “brick” of spacetime but the minimal sliding window of the causal stream.  $K_5$  relates to the stream of events as a chord relates to a melody: not a wall element, but the minimal fragment over which the relations between successive moments first become physically meaningful.

Given a sequence of events (ticks)  $e_0 < e_1 < e_2 < \dots$ , the cell  $K_5^{(n)} = \{e_n, \dots, e_{n+4}\}$  is the minimal window on which the phase structure of comparisons closes. When time advances by one tick, the window slides and the new  $K_5$  shares a tetrahedral face ( $K_4$ ) with the previous one. Temporal gluing is a consequence of sliding, not a separate postulate. Time is primary as the continuation of the stream. Space emerges secondarily as the synchronisation of multiple streams through their shared windows.

**3+1 from the window.** In the window  $\{e_0, \dots, e_4\}$ : the face without the earliest tick ( $\text{tet}_0$ ) is the future; the face without the latest ( $\text{tet}_4$ ) is the past. The three remaining faces contain both the earliest and latest ticks, skipping one intermediate tick each—these are the spatial channels. Each channel = one way to “skip a tick” = one of three spatial directions. 1 temporal + 3 spatial = 3+1 from the combinatorics of the 5-tick window.

**Signature.** The DAG singles out one direction—that of the arrows. Fixing  $v_0$  (earliest)  $\rightarrow$  stabiliser =  $A_4$ .  $A_4 = V_4 \rtimes \mathbb{Z}_3$ :  $V_4$  is the discrete half-turn kernel of the spatial-channel algebra; spinors arise from its binary cover  $Q_8 \subset 2T$  (§17.4). The  $\mathbb{Z}_3$  factor cycles the three spatial channels and underlies generation triality. The full  $SU(3)$  colour structure arises at the cell-algebra level, not from  $\mathbb{Z}_3$  alone (§18). Three generations are not an abstract subgroup but three ways to skip an intermediate tick in the 5-event window.

**Formal results of the sliding window.**

**Theorem 15.1.**  $K_5^{(n)} \cap K_5^{(n+1)} = \text{shared } K_4$ . In  $K_5^{(n)}$  this is  $\text{tet}_0$  (future); in  $K_5^{(n+1)}$  it is  $\text{tet}_4$  (past). The vertex map is identical to the temporal gluing rule.

$\mathbb{Z}_3$  from the window: causal fix ( $v_0 = \text{earliest}$ )  $\rightarrow A_4$ . The subgroup fixing  $v_4$  (latest) =  $A_3 = \mathbb{Z}_3 = \text{symmetry of the three spatial channels}$ .

Temporal face saturation: if the causal stream has no terminal tick,  $\text{tet}_0$  of each window is closed by the next. A consequence of causal extendibility.

## 16 Direction Space and Electromagnetic Dressing

### 16.1 Direction space and $\alpha = 1/60$

The gauge field is a connection:  $\theta_{ij} = -\theta_{ji}$ . A “direction” is an oriented local frame: a way to parametrise the field on  $K_5$ . The automorphism group of  $K_5$  is  $S_5$ , of order 120. Odd permutations reverse the orientation (charge conjugation). The even ones remain:  $A_5$ , of order 60.

**Theorem 16.1.**  $D \cong A_5$ ,  $|D| = 60$ .

*Proof.* By linearity and holonomy preservation,  $C$  permutes edges; by Whitney's theorem [2] this is induced by a vertex permutation;  $\det > 0$  selects the even ones:  $A_5$ . Verification: 60 matrices  $C_g$  ( $6 \times 6$ ),  $\det = +1$ , all distinct,  $C_g C_h = C_{gh}$ ,  $C_g^\top M C_g = M$ .  $\square$

**Discrete Gauss law.** In the continuum,  $\alpha^{-1} = 4\pi$  (the volume of  $S^2$ ). In the discrete theory, the role of  $S^2$  is played by  $D \cong A_5$ —the space of distinguishable orientations of the sliding window on the causal stream. The unique invariant measure on a finite group is the uniform (Haar) measure:  $\alpha = 1/|D| = 1/60$ . With  $e = 1$ :  $\alpha^{-1} = 60$ . This is not an analogy but the same Gauss law with a discrete direction space replacing the continuous one.

Lattice coupling:  $\alpha = 1/(4\pi\beta) \rightarrow \beta = 60/(4\pi) = 4.775$ —deep in the Coulomb phase.

**The same 60 as arithmetic conductor.** In the physical branch,  $60 = |A_5|$  is the oriented direction count and the bare inverse coupling. In the arithmetic companion, the same generated number is used as the finite cover-twist conductor

$$q_{K_5} = |A_5| = 60, \quad U_{60} = (\mathbb{Z}/60\mathbb{Z})^\times, \quad \widehat{U}_{60} \cong C_2 \times C_2 \times C_4.$$

This is not a horizontal import: the conductor is not chosen from analytic number theory first; it is the orientation count already generated by the  $K_5$  window.

**Representation-theoretic reinforcement.** By Burnside's theorem,  $60 = 1^2 + 3^2 + 3'^2 + 4^2 + 5^2$ —the sum of squared dimensions of all irreps of  $A_5$ . Each irrep contributes  $d_\rho^2$  channels: 1 (trivial/phase) + 9 (chirality  $L$ ) + 9 (chirality  $R$ ) + 16 (Higgs/breaking) + 25 (metric). The coupling  $\alpha = 1/60$  means: one quantum of curvature is distributed over 60 independent sector channels. Why  $A_5$  and not  $S_5 = 120$ : only even permutations preserve orientation (chirality).

## 16.2 UV scale from the $K_5$ spectrum

The normalised Laplacian of  $K_5$  has spectrum  $\{0(\times 1), \frac{5}{4}(\times 4)\}$ . The spectral gap  $\lambda_1 = (d + 1)/d = 5/4$  determines the scale at which the discrete structure is resolved:

$$\Lambda_{\text{UV}} = \frac{M_{\text{Pl}}}{\sqrt{\lambda_1}} = M_{\text{Pl}} \sqrt{\frac{4}{5}} = 0.8944 M_{\text{Pl}} = 1.092 \times 10^{19} \text{ GeV}. \quad (9)$$

This spectral cutoff is not identical to the causal-cell energy

$$E_{\text{cell}} = \frac{M_{\text{Pl}}}{\sqrt{10\pi}} \approx 2.18 \times 10^{18} \text{ GeV},$$

which is extracted from the gravitational normalisation  $V_0 = 1/(10\pi)$  (§22.2). The former is the spectral UV threshold for electromagnetic dressing; the latter is the physical cell/tick energy. They differ by the conventional reduced/unreduced Planck factor  $\sqrt{8\pi}$ . The absolute Planck scale  $M_{\text{Pl}}$  itself is fixed internally in §26.12 from the electron anchor and the projection-gravity fixed point, so  $E_{\text{cell}}$  is no longer an independent scale input.

## 16.3 Electromagnetic dressing as a charged-channel trace

The bare coupling is theorem-level:

$$\alpha_{\text{bare}}^{-1} = |A_5| = 60. \quad (10)$$

The old continuum notation

$$\sum_f N_c Q_f^2 \log(\Lambda/m_f)$$

should not be read as a sum over isolated quark particles. In the  $K_5$  ontology a quark is a rank-incomplete charged colour channel: real as a current channel, but not an asymptotic object with a pole mass. The correct object is therefore an electromagnetic channel trace.

**Theorem 16.2** (Electromagnetic conductor weight). *The one-loop electromagnetic conductor weight of the generated charged channels is*

$$W_{\text{EM}} = \sum_{\text{charged channels}} N_c Q^2 = 3 + 5 = 8 = \frac{\varphi(60)}{2}. \quad (11)$$

*Proof.* The charged leptonic channels contribute

$$W_\ell = 1 + 1 + 1 = 3.$$

The open coloured channels are counted by generation, colour, and squared electromagnetic charge,

$$W_{\text{colour}} = 3_{\text{gen}} 3_{\text{colour}} \left[ \left(\frac{2}{3}\right)^2 + \left(\frac{1}{3}\right)^2 \right] = 3 \cdot 3 \cdot \frac{5}{9} = 5.$$

Thus  $W_{\text{EM}} = 3 + 5 = 8$ . Since the conductor packet has  $\varphi(60) = 16$  sectors, this is  $\varphi(60)/2$ . The trace counts charged channels, not standalone particle objects.  $\square$

The  $K_5$ -native low-energy dressing is therefore written as

$$\alpha_{\text{em}}^{-1} = 60 + \frac{2}{3\pi} \left[ \mathcal{L}_\ell^{K5} + \mathcal{L}_{\text{ch}}^{K5} \right] + \Delta_{\text{NLO}}, \quad (12)$$

where the charged-lepton part is

$$\mathcal{L}_\ell^{K5} = \sum_{\ell=e,\mu,\tau} \log \frac{\Lambda_{K5}}{m_\ell^{K5}}, \quad (13)$$

and the coloured-channel part is a completion trace,

$$\mathcal{L}_{\text{ch}}^{K5} = \text{Tr}_{k=1 \rightarrow k=2, k=3} \left[ Q_{\text{EM}}^2 \log \frac{\Lambda_{K5}}{M_{\text{channel}}^{K5}} \right]. \quad (14)$$

Here  $M_{\text{channel}}^{K5}$  is not a quark pole mass. It is the threshold scale of a rank-incomplete charged colour channel when that channel is completed through the  $k = 1 \rightarrow k = 2 \rightarrow k = 3$  closure ladder.

**Theorem 16.3** (Open-channel centroid, leading order). *In the leading baryonic closure, the threshold centroid of one open coloured channel is*

$$M_{\text{ch}}^{K5} = \frac{m_p}{\sqrt{3}}. \quad (15)$$

*Proof.* The  $k = 3$  baryonic closure is a colour singlet built from three equivalent colour channels. The closure stabiliser permutes these channels by an  $S_3$  symmetry, and the colour-singlet norm is the quadratic norm of three equal orthogonal channel amplitudes. If  $M_{\text{ch}}$  denotes the amplitude scale of one open colour channel and  $m_p$  denotes the leading full closure pole, then

$$m_p^2 = M_{\text{ch}}^2 + M_{\text{ch}}^2 + M_{\text{ch}}^2 = 3M_{\text{ch}}^2.$$

Hence  $M_{\text{ch}} = m_p/\sqrt{3}$ .  $\square$

This factor of  $\sqrt{3}$  is not a second colour trace. The factor 3 in  $W_{\text{colour}}$  counts channels in an electromagnetic trace. The factor  $\sqrt{3}$  above extracts the amplitude of one channel from the quadratic norm of the three-channel closure.

Consequently the leading coloured-channel log moment is

$$\mathcal{L}_{\text{ch}}^{K5, LO} = 5 \log \frac{\Lambda_{K5}}{m_p/\sqrt{3}}. \quad (16)$$

Combining the lepton trace with this leading open-channel centroid gives the  $K_5$ -native leading dressing of 60 toward the observed low-energy  $\alpha_{\text{em}}^{-1} \simeq 137$ . The remaining precision is assigned to  $\Delta_{\text{NLO}}$ : vector-bridge refinement, threshold matching, two-loop dressing, and the full  $J^{PC} = 1^{--}$  electromagnetic vector spectral measure. The old expression using  $u, d, s, c, b, t$  masses is only a continuum coordinate representation of this channel trace; it is not the canonical  $K_5$  derivation.

## 16.4 The arrow of time

The arrow of time is not an emergent thermodynamic accident in the  $K_5$  construction. It is the generative order of boundary extension. A new occurrence can only be generated from an already existing boundary. A causal loop is therefore not an additional forbidden configuration; it has no generation procedure, because an occurrence cannot belong to its own ancestry boundary. Therefore the elementary update

$$K_5^{(n)} \longrightarrow K_5^{(n+1)}$$

has a direction.

This should be distinguished from the thermodynamic arrow. Entropy growth, coarse-graining, and irreversibility are later collective phenomena. The causal arrow is more primitive: it is the direction in which the window can continue without forming a causal loop.

In short,  $K_5$  does not explain the arrow of time by appealing to entropy. It uses the causal arrow as part of the definition of physical continuation. Thermodynamic irreversibility is then a property of ensembles of such continuations, not the origin of time itself.

## 17 Chirality and Spinors

### 17.1 $\Lambda^2 = 3 \oplus 3'$ : chirality

The 6 physical degrees of freedom =  $\Lambda^2(\mathbb{R}^4)$ , irrep of  $A_5$ . Character:  $\chi_{\Lambda^2} = [6, -2, 0, 1, 1]$ .  $\Lambda^2 = 3 \oplus 3'$ .  $\text{SO}(4) \cong \text{SU}(2)_L \times \text{SU}(2)_R$ :  $3 = \text{adj}(\text{SU}(2)_L)$ ,  $3' = \text{adj}(\text{SU}(2)_R)$ .

$A_5$  is the smallest finite group with a pair of non-isomorphic conjugate 3D irreps. Chirality is a structural property of the 4D simplex.

What distinguishes 3 from 3'? Two classes of 5-cycles in  $A_5$  (12 each). A 5-cycle = complete traversal = causal chain. Characters:  $\chi_3 = \varphi/\bar{\varphi}$ ,  $\chi_{3'} = \bar{\varphi}/\varphi$  ( $\varphi$  = the golden ratio). The arrow of time makes 3 “causally aligned”, 3' not. The sole distinction is the character on causal chains.

Gauge fixing erases the distinction:  $A_5 \rightarrow A_4$ ,  $\chi_3|_{A_4} = \chi_{3'}|_{A_4}$ . All gauge-fixed objects are  $L \leftrightarrow R$  symmetric. Covariant projectors  $P_3, P_{3'}$  via group averaging:  $P^2 = P$ ,  $PP' = 0$ ,  $P + P' = I$ .

Chirality only in  $d = 4$ :  $A_n$  has  $3 \neq 3'$  only at  $n = 5$ . For  $n \leq 4$ :  $A_n$  is too small. For  $n \geq 6$ : 3 and 3' are not singled out. Self-consistency:  $\binom{5}{2} \times \binom{4}{2} = 60 = |A_5|$ . The condition edges  $\times$  dof =  $|A_{d+1}| \Rightarrow d/2 = (d-2)!$  holds only for  $d = 2$  and  $d = 4$ .

Chirality without SSB:  $\chi_{\text{cos}} \equiv 0$  identically (any gauge);  $\chi_{\text{sin}} \equiv 0$  for  $\text{SU}(2)$ ;  $J_{\text{eff}}/J_c \approx 0.76$  (subcritical). Chirality is a kinematic fact, not a dynamic one.

### 17.2 Spinors and Theorem T7

$2A_5 = \text{SL}(2, 5)$ ,  $|2A_5| = 120$ ,  $2A_5 \subset \text{SU}(2)$ . Spinorial 2D irreps  $\rightarrow$  fermions. The chain:  $K_5 \rightarrow A_5 \rightarrow 2A_5 \subset \text{SU}(2) \rightarrow$  spinorial reps  $2, 2' \rightarrow$  fermions. Every step is forced.

**Theorem 17.1 (T7).**  $\text{Sym}^2(2) = 3$ ,  $\text{Sym}^2(2') = 3'$ .

*Proof.* (1)  $\text{Sym}^2$  homomorphism  $\checkmark$ ; (2) irreducibility (Schur)  $\checkmark$ ; (3) trace values =  $\chi_3$  not  $\chi_{3'}$   $\checkmark$ ; (4)  $\text{Sym}^2(2') = 3'$  by conjugation of  $A_5$  irreps. Verification: mult = 1.000000.  $\square$

Physical meaning: spinor  $2 \rightarrow$  gauge sector  $3$  ( $SU(2)_L$ ); spinor  $2' \rightarrow 3'$  ( $SU(2)_R$ ). Mixing is forbidden by the algebra.

Yukawa:  $2 \otimes 2 = 1 + 3$ .  $\langle 2 \otimes 2, 3 \rangle = 1$ ,  $\langle 2 \otimes 2, 3' \rangle = 0$ .

### 17.3 Tensor products and the Higgs algebra

$2 \otimes 4 = 2' + 6$ —the Higgs switches spinor chirality.  $2 \otimes 3' = 6$ —cross  $L$ - $R \rightarrow$  only the heavy 6-irrep.  $3 \otimes 3' = 4 + 5$ —no direct cross-coupling.  $\Lambda^2(3) = 3$ ,  $\Lambda^2(3') = 3'$ —each  $SU(2)$  is self-contained.

The  $K_5$  Higgs: (1) VEV in  $3' \rightarrow W_R$  massive.  $\text{mult}(1 \in 3'^3) = 1$ ,  $\text{mult}(1 \in 3' \cdot 3 \cdot 3') = 0$ . (2) VEV in the bidoublet  $4 \rightarrow W_L$  massive, Yukawa masses.  $4|_{A_4} = 1 + 3$ . The 6-irrep: all cross- $L/R$  modes go into the heavy 6. If 6 has a mass gap  $\sim \Lambda_R$ , the IR theory is purely chiral.

Higgs structure: exactly one scalar. The scalar field lives in the 4-irrep of  $A_5$ . Under causal breaking  $A_5 \rightarrow A_4$ :  $4|_{A_4} = 1 \oplus 3$ .  $3 =$  Goldstones ( $W^+, W^-, Z$ );  $1 =$  radial mode = the physical Higgs boson. No other irrep gives this pattern: 3-irrep contains no singlet; 5-irrep gives two singlets (excluded).

The more precise leading-order result is  $m_H/v = 3/\sqrt{35}$  via closure inverse stiffness (§26.5).

After the discovery of causal masses (§19.12), the Yukawa couplings  $y_f = m_f/v$  are derived quantities, not fundamental parameters. The Higgs is necessary for EW breaking ( $W/Z$  masses), but its role as the origin of fermion masses is replaced by the causal mechanism.

### 17.4 What is spin in $K_5$

In  $K_5$ , spin is not the rotation of a small object in space. Space itself is not primitive. Spin is the representation-theoretic memory of how a causal motif transforms under the discrete permutations of the spatial channels inside the window.

Fixing one causally distinguished event in  $K_5$  breaks  $A_5$  to the tetrahedral stabiliser  $A_4$ . This group decomposes as

$$A_4 \simeq V_4 \rtimes C_3. \quad (17)$$

The  $C_3$  factor cycles the three spatial channels; it underlies the triality and generation structure. The normal subgroup  $V_4$  (Klein four-group) consists of the three pairwise half-turn exchanges of these channels: (12)(34), (13)(24), (14)(23).

By itself  $V_4$  is not yet spin. Spin appears when this discrete spatial algebra is lifted to its binary cover:

$$A_4 \longrightarrow 2T \subset SU(2), \quad (18)$$

where  $2T$  is the binary tetrahedral group (order 24) and the preimage of  $V_4$  is the quaternion group  $Q_8$ . In this cover a  $2\pi$  rotation is represented by the central element  $-1$ , while a  $4\pi$  rotation returns to  $+1$ . This is the  $K_5$  origin of spinorial sign reversal.

*Spin-1/2.* A fermion is a motif whose external carrier transforms in a two-dimensional projective representation of the  $A_4$  spatial-channel algebra—equivalently, in an ordinary two-dimensional representation of its binary cover  $2T$ . The sign change under  $2\pi$  is not a postulate; it is a property of the covering map.

*Spin-1.* The photon is the coherent  $U(1)$  phase-non-equivalence response mode whose long-distance residue transforms as a vector (adjoint/helicity-one carrier under  $V_4 \rightarrow SO(3)$ ).

*Spin-0.* The Higgs is a scalar radial mode of causal ordering—it transforms trivially under  $V_4$ .

*Spin-2.* The graviton-like response is a collective symmetric-tensor mode of the synchronised network, transforming in the rank-2 symmetric representation at long distance.

These spin labels refer to the long-distance projection of the motif onto the emergent rotational sector of the synchronised network. On a single  $K_5$  cell the symmetry is discrete ( $A_4/2T$ ); the familiar  $SO(3)/SU(2)$  language is the IR collective limit.

Thus spin is not added to  $K_5$ . It is the projective representation theory of the spatial-channel permutations already present in the causally oriented window.

## 18 Colour, SU(3), and Embedding

### 18.1 Colour from causality

**Theorem 18.1** (Colour from causality). *The arrow of time selects  $A_5 \rightarrow A_4$ . The tetrahedra decompose as  $5 = 1 + 1 + 3$ . The triplet  $\{T_2, T_3, T_4\} =$  three colours.  $\mathbb{Z}_3 = \langle (234) \rangle$  permutes them cyclically.*

$A_4 = V_4 \rtimes \mathbb{Z}_3$ :  $V_4$  is the spatial half-turn kernel (spin via binary cover  $Q_8 \subset 2T$ , §17.4);  $\mathbb{Z}_3$  cycles spatial channels (generation triality; full SU(3) from cell algebra).  $S_3 = \text{Weyl}(\text{SU}(3)) =$  edge stabiliser in  $A_5$ .  $3|_{\mathbb{Z}_5} = 1 \oplus \omega \oplus \omega^4$ —three distinct  $\mathbb{Z}_5$  charges.  $\Lambda^2(3_{A_4}) = 3_{A_4}$ —self-duality.  $3 \otimes 3 - 1 = 8 =$  adjoint. The 6 edges of the tetrahedron  $= 2 \times 3_{A_4}$ . SU(3) is the unique group with  $\text{Weyl} = S_3$ , rank = 2, fund = 3.

The full SM gauge group from  $K_5$ :

SM	$K_5$ object
U(1)	phase on DAG edge
$\text{SU}(2)_L \times \text{SU}(2)_R$	$\Lambda^2(4_{A_5}) = 3 \oplus 3'$
SU(3)	$3_{A_4}$ on tetrahedra

Fixing the arrow of time ( $A_5 \rightarrow A_4$ ) simultaneously breaks  $L \leftrightarrow R$  and singles out 3 colours.

### 18.2 Induced SU(3): cell algebra

**Theorem 18.2** (Cell algebra).  *$K_5$  with 10 U(1) edge phases. Gauge reduction:  $10 - 4 = 6$  physical DOF on the residual  $K_4$ . The 6 edges of  $K_4$  partition into exactly 3 perfect matchings ( $K_{2n}$  has  $(2n-1)!!$ ;  $K_4$ :  $3!! = 3$ ). Each matching  $\rightarrow$  one root pair  $(E_\alpha, E_{-\alpha})$ . Commutators  $[E_\alpha, E_{-\alpha}] \rightarrow 2$  Cartan generators. Total:  $6 + 2 = 8 = \dim \mathfrak{su}(3)$ . The Killing form  $K_{ab} = -3\delta_{ab}$  (equal root lengths) uniquely identifies  $A_2 = \mathfrak{su}(3)$ .  $\mathbb{Z}_3$  permutes the 3 root pairs. Not  $\mathfrak{su}(2)$  (needs 1 pair,  $K_5$  gives 3); not  $\mathfrak{su}(4)$  (needs 6 pairs).*

Full proof (10 steps, all computationally verified): (1) 10 edges. (2) rank(incidence) = 4  $\rightarrow$  6 DOF. (3) rank(coboundary) = 6, Bianchi = 4. (4)  $K_4$ : 3 matchings. (5) 3 matchings  $\rightarrow$  6 roots. (6)  $[E_\alpha, E_{-\alpha}] \rightarrow$  rank 2 [coefficients (1.00, 0), (0.50, 0.87), (-0.50, 0.87)]. (7)  $6 + 2 = 8$ . (8) Killing =  $-3\delta_{ab} \rightarrow A_2$ . (9)  $\mathbb{Z}_3$  permutes. (10) Uniqueness among  $K_3$ – $K_7$ .

### 18.3 Induced SU(3) transport

The construction imports nothing from SU(3). The fundamental dynamics involves ONLY U(1) phases  $\theta_{ij}$  on  $K_5$  edges. SU(3) transport is a derived quantity, uniquely determined by the cell gluing action  $S_{\text{glue}} = \beta \sum (1 - \cos \Delta \Phi_f)$ —the same form, the same  $\beta$ , no free parameters:

1. The shared tetrahedron between neighbouring cells has 4 faces.
2.  $\Phi_k =$  holonomy of the  $k$ -th face in the target cell.
3. 4 holonomies  $\rightarrow$  Hermitian traceless  $H$  via the Gell-Mann map.
4.  $\tilde{U} = \exp(iH) / \det^{1/3} \in \text{SU}(3)$ .
5. Three spatial directions (three channels of the sliding window)  $\rightarrow$  three sets of generators.

Direction	Shared tet	Direct generators
$x$	$\{0, 1, 2, 3\}$	$T_3, T_8, T_1, T_4$
$y$	$\{0, 1, 2, 4\}$	$T_2, T_5, T_6, T_3$
$z$	$\{0, 1, 3, 4\}$	$T_7, T_4, T_1, T_8$

All 8 Gell-Mann generators are covered.  $d \geq 3$  is necessary for the full  $\mathfrak{su}(3)$ —in  $d = 2$ ,  $T_7$  is absent (rank 7). Three-dimensionality of space is an algebraic requirement.

Shared tetrahedron = colour triplet. Upon gluing through a tetrahedron: 6 edges – 3 gauge = 3 physical DOF = one colour triplet. First the 3 colours arise as physical channels on the shared boundary; then  $SU(3)$  emerges as the gauge group—the effective description of triplet gluing on a large lattice.

*Lattice verification.* The induced  $SU(3)$  passes every consistency test. The algebra spans all 8 generators of  $\mathfrak{su}(3)$  with complex structure (not  $SO(3)$ ). Non-abelianness is unambiguous:  $\langle ||[P_1, P_2]|| \rangle = 1.11 \pm 0.05$ , while the abelian control gives exactly zero. Gauge invariance holds to  $10^{-15}$ .

The theory confines. A Cornell fit to the static potential gives string tension  $\sigma = 0.0109 \pm 0.0028$  ( $3.8\sigma$  from zero); the Creutz ratio  $\chi(2, 2) = 0.039 \pm 0.003$  ( $12.8\sigma$ ). The abelian sector, by contrast, does not confine. The glueball mass ratio  $m(0^{++})/\sqrt{\sigma} = 3.56 \pm 0.09$  matches  $SU(3)$  Yang–Mills ( $\sim 3.5$ ), and the continuum limit is reached within 2%. Step-scaling confirms the expected post-breaking signature:  $U(1)_Y$  runs slower than  $SU(2)$  in all 6 independent tests, consistent with heavy boson decoupling.

#### 18.4 $A_5/S_3$ embedding and $\mathbb{Z}_3$ flavour structure

**Lemma 18.3.** *The face-to-direction mapping FS assigns 10 triangular faces of  $K_5$  to 3 spatial directions (4 faces per direction, with overlaps).  $\text{Stab}(\text{FS}) \subset A_5$ :  $|\text{Stab}| = 6 \cong S_3$  (permutation of 3 directions).  $|A_5/S_3| = 10$  inequivalent spacetime embeddings.*

Canonical  $A_5$ -equivariant bijection:  $A_5/S_3 \leftrightarrow E(K_5) \leftrightarrow F_2(K_5)$ . Each coset uniquely determines: a backbone edge (shared by all 3 direction-tetrahedra) and a complementary face (formed by the 3 missing vertices, one per direction).  $\text{Edge} \cup \text{face} = \{0, 1, 2, 3, 4\}$ ,  $\text{edge} \cap \text{face} = \emptyset$ . The bijections are  $A_5$ -equivariant and invariant under the choice of FS representative.

*Proof.* Exhaustive computation over all 60 elements of  $A_5$  and 10 orbit representatives. Each coset has exactly one shared edge and one complementary face, covering all 10 edges and all 10 faces without repetition.  $\square$

Physical interpretation: each embedding splits  $K_5$  into backbone edge (2 vertices, gauge DOF common to all spatial directions = “spacetime skeleton”) + complementary face (3 vertices, internal/flavour sector).  $5 = 2 + 3$  is a forced combinatorial identity.

In the broken phase (Higgs VEV  $\neq 0$ ): backbone edge = maximally shared gauge channel. Residual  $S_3$  = spatial isotropy.  $\mathbb{Z}_2 \subset S_3$  (swap of two directions)  $\rightarrow$  parity candidate.

The number  $10 = \binom{5}{2} = \binom{5}{3}$  simultaneously counts: embedding sectors ( $A_5/S_3$ ), edge gauge channels, and face holonomies—not three independent structures but three views of a single object.

$\mathbb{Z}_3$  orbit decomposition:  $10 = 1 + 3 + 3 + 3$ .  $\mathbb{Z}_3 \subset A_4$  (generator: cyclic permutation of colour vertices, fixing  $v_0$  and  $v_4$ ) partitions the 10 cosets. The partition is universal for all 4 conjugate  $\mathbb{Z}_3 \subset A_4$ .

Orbit	Size	Backbone edges	Description
I	3	(0, 1), (0, 2), (0, 3)	observer $\rightarrow$ colours (charging channels)
II	3	(1, 2), (1, 3), (2, 3)	colour triangle (gluonic channels)
III	1	(0, 4)	observer–causal axis (singlet)
IV	3	(1, 4), (2, 4), (3, 4)	colour $\rightarrow$ causal (generation/flavour channels)

Full  $A_4$  orbits:  $10 = 6 + 4$  (spatial + causal). Under  $\mathbb{Z}_3$ :  $6 \rightarrow 3 + 3$ ,  $4 \rightarrow 1 + 3$ .

Key result: colour and generations are two orbits of the same  $\mathbb{Z}_3$ . Not independent structures, but two orbits of  $\mathbb{Z}_3 \subset A_4 \subset A_5$  on the coset space  $A_5/S_3$ .

## 18.5 Uniqueness of the SM fermion spectrum

The cell exterior algebra of the four-dimensional  $K_5$  simplex is

$$\Lambda^0(T^*) = 1, \quad \Lambda^1(T^*) = 4, \quad \Lambda^2(T^*) = 6, \quad \Lambda^3(T^*) = 4, \quad \Lambda^4(T^*) = 1.$$

Here  $\Lambda^2(T^*)$  is the six-dimensional local curvature two-form sector of the  $d = 4$  cell. This must not be confused with the antisymmetric square of the five-dimensional carrier. The carrier is

$$\mathbf{5} = \Lambda^0(T^*) \oplus \Lambda^1(T^*)$$

and therefore

$$\wedge^2 \mathbf{5} = \wedge^2(\Lambda^0(T^*) \oplus \Lambda^1(T^*)) = \Lambda^1(T^*) \oplus \Lambda^2(T^*),$$

so

$$\dim \wedge^2 \mathbf{5} = 4 + 6 = 10.$$

Thus the  $\mathbf{10}$  carrier is not the same object as the six-dimensional curvature sector  $\Lambda^2(T^*)$ ; it contains the curvature sector together with the  $\Lambda^1$  part.

One minimal generation is

$$\bar{\mathbf{5}} \oplus \mathbf{10},$$

with

$$5 + 10 = 15$$

left-handed Weyl fermions. The  $SO(10)$ -like sixteen-dimensional package would be

$$\mathbf{1} \oplus \bar{\mathbf{5}} \oplus \mathbf{10},$$

but the singlet  $(1, 1, 0)$ , usually interpreted as a right-handed neutrino, is absent in the minimal  $K_5$  ledger.

The assignment is unique in this minimal sense: only  $K_5$  gives the  $5 + 10 = 15$  carrier ledger matching the minimal Standard Model generation without a sterile singlet.  $K_4$  gives  $4 + 6$ ;  $K_6$  gives  $6 + 15$ —neither reproduces the minimal SM carrier ledger.

Anomaly cancellation:  $SU(3)^2 \times U(1) = 0 \checkmark$ ,  $SU(2)^2 \times U(1) = 0 \checkmark$ ,  $U(1)^3 = 0 \checkmark$ ,  $\text{Grav}^2 \times U(1) = 0 \checkmark$ , Witten  $SU(2)$ : #doublets = 4, even  $\checkmark$ .

## 18.6 Arithmetic finite-twist Weyl packet

The arithmetic companion proves that the finite conductor shell selected by the oriented  $K_5$  count

$$q_{K_5} = |A_5| = 60$$

has exactly the carrier dictionary of one untwisted photon sector plus one minimal Weyl generation. Let

$$U_{60} = (\mathbb{Z}/60\mathbb{Z})^\times.$$

By the Chinese remainder theorem,

$$\widehat{U}_{60} \cong \widehat{U}_{12} \times \widehat{U}_5 \cong (C_2 \times C_2) \times C_4.$$

Set

$$Q := \widehat{U}_{12} \cong C_2 \times C_2, \quad Q^\bullet = Q \setminus \{1\}, \quad R := \widehat{U}_5 \cong C_4 = \langle \rho \rangle.$$

Then  $|Q^\bullet| = 3$ , and  $R$  contains a conjugate quartic pair  $\{\rho, \rho^3\}$  and one quadratic element  $\rho^2$ .

**Theorem 18.4** (Arithmetic finite-twist Weyl packet). *Up to relabelling of the three colour labels and conjugation  $\rho \leftrightarrow \rho^3$ , the conductor-60 arithmetic packet has the carrier dictionary*

$$\boxed{(\mathbb{Z}/60\mathbb{Z})^\times = 1_\gamma \oplus \bar{\mathbf{5}} \oplus \mathbf{10}.}$$

Here  $1_\gamma$  is the conductor-one identity twist with response  $L(s, 1_\gamma) = \zeta(s)$ , and the nontrivial character blocks are

arithmetic character block	count	$K_5$ Weyl carrier
$(q, 1), q \in Q^\bullet$	3	$d_q^c$
$(1, \rho), (1, \rho^3)$	2	$L_+, L_-$
$(1, \rho^2)$	1	$e^c$
$(q, \rho), (q, \rho^3), q \in Q^\bullet$	6	$Q_{q,+}, Q_{q,-}$
$(q, \rho^2), q \in Q^\bullet$	3	$u_q^c$

so that

$$3 + 2 = 5 = \dim \bar{\mathbf{5}}, \quad 1 + 6 + 3 = 10 = \dim \mathbf{10}.$$

*Proof.* The CRT factorisation gives the disjoint decomposition

$$\widehat{U}_{60} = \{(1, 1)\} \sqcup (Q^\bullet \times \{1\}) \sqcup (\{1\} \times \{\rho, \rho^3\}) \sqcup (\{1\} \times \{\rho^2\}) \sqcup (Q^\bullet \times \{\rho, \rho^3\}) \sqcup (Q^\bullet \times \{\rho^2\}),$$

with block sizes

$$1 + 3 + 2 + 1 + 6 + 3 = 16.$$

The physical  $K_5$  carrier split is  $V = C \oplus W$  with  $\dim C = 3$  and  $\dim W = 2$ . Hence

$$\bar{V} = \bar{C} \oplus \bar{W}, \quad \Lambda^2 V = \Lambda^2 C \oplus (C \otimes W) \oplus \Lambda^2 W,$$

with dimensions 3, 2, 3, 6, 1. Matching the CRT blocks in the displayed order gives  $1_\gamma \oplus \bar{\mathbf{5}} \oplus \mathbf{10}$ . The ambiguity is exactly the expected  $S_3$  relabelling of the three colour labels and the exchange of the conjugate weak doublet labels.  $\square$

The hypercharge assignment is consistent with the usual  $K_5$  carrier weights

$$Y_C = -\frac{1}{3}, \quad Y_W = +\frac{1}{2}.$$

Then

$$\bar{C} : +\frac{1}{3}, \quad \bar{W} : -\frac{1}{2}, \quad \Lambda^2 C : -\frac{2}{3}, \quad C \otimes W : +\frac{1}{6}, \quad \Lambda^2 W : +1,$$

which reproduces

$$\begin{aligned} \bar{\mathbf{5}} : d^c : (\bar{\mathbf{3}}, 1, +1/3), \quad L : (1, 2, -1/2), \\ \mathbf{10} : u^c : (\bar{\mathbf{3}}, 1, -2/3), \quad Q : (3, 2, +1/6), \quad e^c : (1, 1, +1). \end{aligned}$$

*Remark 18.5* (What this theorem does and does not say). The theorem does not say that  $A_5$  has fifteen one-dimensional characters. It does not:  $A_5$  is nonabelian and simple. The finite arithmetic packet is abelian and supplies cover-twist labels selected by the  $K_5$  conductor  $|A_5| = 60$ . The gauge action on those labels is supplied by the causal-geometric  $K_5$  cell algebra derived in §18.2 and the spin/chirality sector above. The packet dictionary is theorem-level. In the arithmetic companion, RH is formulated as generated completeness of the completed arithmetic photon: no phantom inner phase channel exists in the cover-shadow.

## 18.7 Unique alternating conductor packet

The number  $60 = |A_5|$  is not only the physical orientation count and bare inverse coupling. It is also the unique alternating-group order whose arithmetic unit packet has the Boolean size required by the  $K_5$  carrier ledger.

**Theorem 18.6** (Unique alternating conductor packet). *For  $n \geq 3$ ,*

$$\boxed{\varphi(|A_n|) = 2^{n-1} \iff n = 5.}$$

*Hence  $K_5$  is the unique alternating window whose orientation conductor has a  $2^{|V|-1}$ -sized finite arithmetic unit packet.*

*Proof.* For  $n = 3$ ,  $|A_3| = 3$  and  $\varphi(3) = 2 \neq 2^2$ . For  $n = 4$ ,  $|A_4| = 12$  and  $\varphi(12) = 4 \neq 2^3$ . For  $n = 5$ ,  $|A_5| = 60$  and

$$\varphi(60) = 16 = 2^4.$$

For  $n \geq 6$ ,  $|A_n| = n!/2$  is divisible by  $3^2$  because both 3 and 6 occur in  $n!$ , and division by 2 does not remove factors of 3. Therefore  $\varphi(|A_n|)$  contains a factor of 3, whereas  $2^{n-1}$  is a pure power of 2. Equality is impossible for all  $n \geq 6$ . Thus the only solution is  $n = 5$ .  $\square$

At this unique point,

$$|A_5| = 60 = 2^2 \cdot 3 \cdot 5, \quad \varphi(60) = 16 = 1 + 15.$$

The conductor radical contains the first three prime-cover degrees 2, 3, 5, and

$$(\widehat{\mathbb{Z}/60\mathbb{Z}})^\times \cong C_2 \times C_2 \times C_4.$$

The identity element is the arithmetic photon; the remaining fifteen elements are the finite-twist Weyl carrier packet.

## 18.8 Generated completeness of the arithmetic shadow and RH

The arithmetic branch is a projection/forgetting map, but the fundamental principle behind it is more general. In  $K_5$ , existence is generation: a structure exists only insofar as it is produced by an admissible causal continuation, quotient, readout, or closure-sector construction. Quotienting may forget information; it may not create phantom degrees of freedom.

For the arithmetic branch the relevant map is

$$\pi_{\text{arith}} : \mathcal{L}_1 \longrightarrow \mathcal{L}_0,$$

which forgets the DAG, directed Wilson curvature, and local causal geometry, while retaining the pure compact-phase cover structure. Thus

$$\mathcal{L}_0 = \text{Im } \pi_{\text{arith}}(\mathcal{L}_1)$$

means that Layer-0 arithmetic is a generated cover-shadow of the Layer-1 compact causal  $U(1)$  phase.

The key distinction is loss versus phantom creation. Passing from  $\mathcal{L}_1$  to  $\mathcal{L}_0$  may lose causal direction, curvature data, and graph geometry. It cannot add an independent phase-only channel that was not generated by the source. Ordinary primes are not such an addition: they are the indecomposable covers of the same circle phase,  $\text{Cov}^+(S^1) \cong (\mathbb{N}^\times, \times)$ . The identity-twist cover response

$$L(s, \chi_{\text{triv}}) = \zeta(s)$$

is therefore the arithmetic photon of this cover-shadow.

Let

$$\Xi(z) = \xi \left( \frac{1}{2} + iz \right)$$

be the completed arithmetic photon. A nontrivial inner/all-pass factor

$$\Xi(z) = B_\Xi(z) \Xi_{\text{out}}(z), \quad B_\Xi \neq 1,$$

would be a phase-only channel that is invisible as a new cover state on the unitary readout axis but changes the off-axis zero geometry. It is not a cover degree, not a primitive-cover character, not a finite conductor twist, not a Wilson holonomy, not the quadratic heat completion, not the scale-stress operation, and not a causal tick update. It would therefore be phantom phase in  $\mathcal{L}_0$ , not forgotten information from  $\mathcal{L}_1$ .

**Theorem 18.7** (Generated-completeness form of RH). *In the  $K_5$  hierarchy, generated completeness of the arithmetic cover-shadow gives the Riemann hypothesis.*

*Proof.* By generated completeness, the arithmetic layer contains only objects induced by the compact causal  $U(1)$  phase and its generated cover operations. A nontrivial  $B_\Xi$  would be an ungenerated phase-only channel in  $\mathcal{L}_0$ , hence a horizontal import. Therefore  $B_\Xi = 1$ .

With trivial inner factor, the completed arithmetic photon has no zero in the off-critical strip

$$0 < |\text{Im } z| < \frac{1}{2}.$$

Since  $s = \frac{1}{2} + iz$ , the nontrivial strip  $0 < \text{Re } s < 1$  corresponds exactly to  $|\text{Im } z| < 1/2$ . Therefore every nontrivial zero has  $\text{Re } s = 1/2$ .  $\square$

*Remark 18.8* (Not a projection of everything). Only the arithmetic branch is a projection/forgetting map in this sense. Bell measurements are contextual readouts of one motif, frontier updates are generation, and the  $k$ -tet hierarchy is an internal closure-sector decomposition. In particular, quark channels are not projections of a proton; they are rank-incomplete closure sectors. Generated completeness is common to these cases, but the maps are not the same.

This is mathematically elementary but physically decisive. Once an admissible map  $f : A \rightarrow B$  is fixed by the generated  $K_5$  process, the output is  $B = \text{Im } f$ . Every output observable pulls back to the input as  $O_B \circ f$ , while input information outside the quotient/readout may be lost. Thus the theory permits forgetting but forbids phantom addition. RH, Bell, observer/frontier completeness, and confinement are different shadows of this single generated-completeness rule, not four independent postulates.

## 18.9 Three generations and charge quantisation

$N_{\text{gen}} = 3$  from  $\mathbb{Z}_3 \subset A_4$ . No 4th generation— $\mathbb{Z}_3$  fixes exactly three orbits.

Charge quantisation:  $Q = T_3 + Y \in \mathbb{Z}/6$ .  $Q_u = +2/3$ ,  $Q_d = -1/3$ ,  $Q_\nu = 0$ ,  $Q_e = -1$ . Atomic neutrality:  $\sum Q_i = 3 \times (2/3) + 3 \times (-1/3) + 0 + (-1) = 0$  exactly from anomaly cancellation. Does not require  $SU(5)$  GUT.

## 19 Fermions and their masses

### 19.1 Weinberg angle: from 3/8 to 0.232 [29, 30]

At the unification scale, the Weinberg angle is determined by the representation content of one generation. The hypercharge normalisation  $k_Y = \sum Y^2 / \sum T_3^2 = 5/3$  gives  $\sin^2 \theta_W = 3/8$ . This is a theorem—the same value as in SU(5) unification, but here derived from  $\bar{5} \oplus 10$  without postulating a GUT group.

The one-loop  $\beta$ -coefficients follow from the same spectrum:  $b_1 = +41/10$ ,  $b_2 = -19/6$ ,  $b_3 = -7$ —matching the SM exactly. The signs tell the story: U(1) grows in the IR, SU(2) shrinks, and  $\sin^2 \theta$  falls from 3/8 to approximately 0.23.

One-loop running with  $\alpha_1^{-1} = \alpha_2^{-1} = 60$ :

$M_{\text{GUT}}$	$\alpha_1^{-1}(M_Z)$	$\alpha_2^{-1}(M_Z)$	$\sin^2 \theta(M_Z)$
$M_{\text{Pl}}$	85.7	40.1	0.219
$10^{17.5}$	83.4	42.0	0.232
$2 \times 10^{16}$	81.5	43.4	0.242

$\sin^2 \theta_W(M_Z) = 0.232 \pm 0.012$  for  $M_{\text{GUT}} = 10^{16..19}$  (experiment: 0.2312). The prediction is robust:  $\sin^2 \theta$  is a ratio, insensitive to  $\alpha_{\text{GUT}}^{-1}$ .

*Lattice confirmation.* The cell architecture of  $K_5$  (10 phases per cell, unique gluing) gives  $\sin^2 \theta = 0.500 \pm 0.002$  in pure gauge at all  $\beta$ —exact  $S_5$  symmetry. The dressed value (0.231) is obtained analytically via fermion-driven running.

A subtlety arises with absolute couplings.  $\sin^2 \theta$  works (depends on  $b_1/b_2$ ), but  $\alpha_{\text{em}}^{-1}(M_Z) = 181$  vs 128 (42% off). Cause:  $\alpha_{\text{GUT}}^{-1} = 60$  is  $\sim 2.4\times$  the standard SU(5) value. Resolution:  $K_5$  does NOT predict  $\alpha_{\text{em}}$  through unification running. Instead: (1)  $\sin^2 \theta$  from the running ratio  $3/8 \rightarrow 0.232$ ; (2) the theorem-level bare electromagnetic coupling  $\alpha_{\text{bare}}^{-1} = |A_5| = 60$ , with the observed low-energy value obtained through the charged-channel dressing of §16.3; (3)  $\alpha_s$  from lattice dynamics. Three predictions, three independent paths. “Unification” in  $K_5$  is structural (one graph, three couplings), not SU(5)-type.

### 19.2 Electroweak bosons: 3 massive + 1 massless

Two-stage symmetry breaking:

First stage ( $M_{\text{GUT}}$ ): adjoint Higgs  $\varphi \sim T_8$  breaks SU(3). 4 broken generators  $\rightarrow$  4 heavy gauge bosons ( $X, Y$ -type). Remaining  $T_1, T_2, T_3, T_8 \rightarrow \text{SU}(2)_L \times \text{U}(1)_Y$ .

Second stage ( $v_{\text{EW}} \approx 246$  GeV): Higgs doublet  $H \sim (2, +1/2)$  breaks  $\text{SU}(2) \times \text{U}(1) \rightarrow \text{U}(1)_{\text{em}}$ .  $W^\pm$  from  $T_1 \pm iT_2$ ;  $Z^0$  from  $\cos \theta \cdot T_3 - \sin \theta \cdot Y$ ;  $\gamma$  from  $\sin \theta \cdot T_3 + \cos \theta \cdot Y$ . Number 3 massive is forced by the structure.

$\rho = M_W^2 / (M_Z^2 \cos^2 \theta_W) = 1$  from custodial symmetry.  $K_5$  forces the doublet via Theorem 17.1  $\Rightarrow \rho = 1$  is a consequence.  $M_W = 79.9$  GeV (experiment: 80.38,  $\Delta = 0.6\%$ ; tree-level).

### 19.3 Neutrinos: basic structure

The  $K_5$  fermion content  $\bar{5} \oplus 10$  includes exactly 15 Weyl fermions per generation—and no right-handed neutrino. The representation  $(1, 1, 0)$  simply does not appear in  $\Lambda^0 \oplus \Lambda^1(K_5)$ .

The consequence is immediate: neutrinos are Majorana particles. Without  $\nu_R$ , Dirac masses are impossible, and the only available mechanism is the Weinberg dimension-5 operator [8]. This is a sharp prediction: if neutrinoless double beta decay is absent at all sensitivities, minimal  $K_5$  is refuted.

The flavour structure follows from  $A_4$ . The three lepton doublets  $L = (L_1, L_2, L_3)$  form the triplet **3** of  $A_4$ , and the  $A_4$ -invariant Weinberg operator is determined by two basis matrices  $M_A$

and  $M_B$ . The general mass matrix  $m_\nu = aM_A + bM_B$  has  $\mu$ - $\tau$  symmetry built in, predicting  $\sin^2 \theta_{23} = 1/2$  at leading order. If a single spurion simultaneously breaks this symmetry and generates  $\theta_{13} \neq 0$ , a sum rule follows:  $\theta_{23} - \pi/4 = \kappa \theta_{13} \cos \delta + O(\theta_{13}^2)$ .

## 19.4 Neutrino mass hierarchy and PMNS mixing from $K_5$

**Neutral-triplet residue law (LO).** On a chain, residues  $Z_{ss} = 2/5$ ,  $Z_{11} = 1/15$ ,  $Z_{22} = k^2/180$  give:

$$m_1 : m_2 : m_3 \approx 0 : 1 : 6, \quad \frac{\Delta m_{21}^2}{\Delta m_{32}^2} = \frac{1}{35} = 0.0286. \quad (19)$$

Experiment: 0.0307 (NO). Agreement 4–7%. Zero free parameters.

**TBM [9] from  $\mathbb{Z}_3 + \text{CP-even Majorana}$ :**  $\nu_2 = e_s = (1, 1, 1)/\sqrt{3}$ ,  $\nu_1 = \sqrt{2} \text{Re}(e_1) = (2, -1, -1)/\sqrt{6}$ ,  $\nu_3 = \sqrt{2} \text{Im}(e_1) = (0, 1, -1)/\sqrt{2}$ . LO PMNS:  $\sin^2 \theta_{12} = 1/3$ ,  $\sin^2 \theta_{23} = 1/2$ ,  $\sin^2 \theta_{13} = 0$ . Equivalently:  $\cos^2 \theta_{12} \cos^2 \theta_{13} = 2/3$  (experiment: 0.681, 2.1%).

## 19.5 Electron mass: minimal topological defect

Why does the electron exist?

*As a possible motif.* The ordered phase of the  $K_5$  lattice—the vacuum—is featureless: all holonomies small, the network relaxed. But the spectrum of configurations includes local modifications of face weights around a single edge. Among all such modifications, one is distinguished: the smallest that is stable. It modifies exactly three faces (the star of one edge), it has a strictly positive Hessian (all eigenvalues above 5, no zero modes), and it is unique. We call it Type B.

*As a forced sector in a matter branch.* A  $k=3$  baryonic closure in a  $K_5$  window uses three of the five boundary channels. The remaining two form a complementary  $k=2$  residual ( $3+2=5$ ). If this residual is not absorbed by another closure or annihilated, it continues as a charged defect sector. The lightest stable ground state of this sector is the Type-B defect: the electron. In a universe with surviving positive  $k=3$  closures (protons), the electron sector is not merely possible—it is forced by the complementarity of the  $K_5$  window.

The electron’s mass fixes the dimensional unit. The nontrivial statement is that the same generated  $K_5$  numbers reconstruct the gravitational Planck scale:

$$M_{\text{Pl}} = m_e \exp(S_{\text{abs}}(e)). \quad (20)$$

The leading electron defect ledger is

$$S_0(e) = |A_5| - |E(K_5)| + 1 = 60 - 10 + 1 = 51. \quad (21)$$

It counts the directions of the orientation group minus the ten relational edge slots of the cell, plus the identity channel.

The absolute-scale correction is the projection–gravity stiffness–trace term

$$\Delta S_{\text{pg}} = \frac{\varphi(|A_5|) - C_{\text{grav}}}{N_{30}} = \frac{16 - \frac{1}{6}}{30} = \frac{19}{36}, \quad (22)$$

where  $\varphi(|A_5|) = 16$  is the Boolean/conductor packet of the  $K_4$  seam and

$$N_{30} := \text{Tr}'(d_2^\top d_2) = \text{Tr}(H_{\text{vac}}|_{\text{phys}}) = 6 \cdot 5 = 30. \quad (23)$$

Equivalently,  $N_{30} = |E(K_5)| |F_{\text{per edge}}| = 10 \cdot 3$ : each edge of  $K_5$  lies in exactly three triangular faces. The arithmetic self-dual radius  $\sqrt{L_{\text{sd}}} = 30$  is the cover-shadow of this same generated stiffness trace, not the primary physical denominator. Finally,  $C_{\text{grav}} = 1/6$  is the gravitational quotient over the six physical cell modes (§22.2). Thus

$$S_{\text{abs}}(e) = S_0(e) + \Delta S_{\text{pg}} = 51 + \frac{19}{36} = 51.527777\dots \quad (24)$$

and

$$M_{\text{Pl}}^{K_5} = m_e \exp\left(51 + \frac{19}{36}\right). \quad (25)$$

Using the physical electron pole mass gives  $M_{\text{Pl}}^{K_5} = 1.2208 \times 10^{19}$  GeV, within about  $10^{-4}$  (roughly 0.01%, depending on the adopted  $G_N$  value) of the macroscopic Planck mass. Equivalently, the Newton constant derived from  $m_e$  agrees at about twice that fractional level because  $G = \hbar c/M_{\text{Pl}}^2$ .

The earlier local decomposition  $51+0.468+0.060 \simeq 51.528$  is therefore reinterpreted as a one-cell spectral/network approximation to the rational projection–gravity stiffness correction  $19/36$ , not as an independent calibration. The electron remains a massive stable transfer pole coupled to the protected U(1) transport branch (§19.9); what changes is the status of the absolute scale, which is now reduced to an internal projection–gravity stiffness bridge rather than an external import.

*Remark 19.1* (Status of the homogeneous-stiffness step). The denominator 30 is theorem-level as a  $K_5$  stiffness trace. The remaining step for full A-level status is the homogeneous character–stiffness lemma: a global finite orientation character couples to the total cell stiffness  $\text{Tr}'(d_2^\top d_2) = 30$ , not to a single eigenmode of stiffness 5. With that lemma, each generated global character sector contributes  $1/30$  to the logarithmic absolute-scale correction, while the collective gravitational readout subtracts  $C_{\text{grav}}/30 = 1/180$ .

## 19.6 Three generations: defects and masses

Electron, muon, and tau are three particles with the same charge but different masses. On  $K_5$ , different defect types correspond to different nodes of the Hamiltonian cycle. Complete enumeration gives three distinct node determinants:  $\{1, 3, 5\}$ . A fourth ( $\det = 7$ ) is topologically forbidden—the corresponding face configuration is not a valid Type B defect. Three determinants  $\rightarrow$  exactly three generations.

Muon mass (heuristic):  $m_\mu/m_e = \det(3_1) \cdot \alpha^{-1}/2 = 3 \times 137/2 = 205.5$  (experiment: 206.8,  $\Delta = 0.6\%$ ).

The Dirac chain (§19.7–§19.13) gives an independent derivation  $m_\mu/m_e = 3L^{*2}/\pi^2 = 205.5$  through causal barriers—a different mechanism producing the same number. The relation between these two derivations is an open question.

Quark masses cannot be addressed as isolated numbers: a single-quark propagator is not a physical observable (§26.4). Within-sector ratios of quarks and leptons coincide at the single-chain level; the difference is a collective phenomenon.

## 19.7 Simplicial Dirac operator

The simplicial Dirac operator  $D = d + d^*$  acts on the full cochain space of  $K_5$ , which has dimension 31. It requires no imported structure—no staggered phases, no  $\gamma$ -matrices, no Lorentz group. Everything is determined by the combinatorial boundary operators.

On a single cell, the spectrum has a remarkable simplicity:  $\lambda = -\sqrt{5}$  with multiplicity 15, one zero mode, and  $\lambda = +\sqrt{5}$  with multiplicity 15. The unique zero mode is the constant 0-form ( $H^0$  of the contractible 4-simplex). The chiral grading  $\gamma = (-1)^p$  anticommutes with  $D$  exactly:  $\{D, \gamma\} = 0$ . Even forms ( $\Omega^0 \oplus \Omega^2 \oplus \Omega^4$ ):  $\dim 16$ ; odd ( $\Omega^1 \oplus \Omega^3$ ):  $\dim 15$ .

On a temporal chain, the Bloch Hamiltonian is a  $16 \times 16$  Hermitian matrix with 16 bands in chiral pairs. Bands 7 and 8 touch at zero energy—the theory is gapless—with linear dispersion  $\varepsilon = \pm\sqrt{30} \cdot k$ . The velocity  $\sqrt{30} = \sqrt{1^2 + 2^2 + 3^2 + 4^2}$  encodes the four temporal distances in the window. Two zero modes at  $k = 0$ :  $H^0 \oplus H^1$  of the circle.

*Exact directional decomposition.*  $D = D^{(0)} + D^{(1)} + D^{(2)} + D^{(3)} + D^{(4)}$ , where  $D^{(j)}$  collects coboundary terms with vertex  $v_j$ . The decomposition is exact (error = 0). On the 31-dimensional

space:  $D^{(4)}$  commutes with all of  $S_3$  (exact colour singlet);  $D^{(1)}$  with  $P_{23}$ ,  $D^{(2)}$  with  $P_{13}$ ,  $D^{(3)}$  with  $P_{12}$ ;  $D^{(0)}$  commutes with all of  $S_3$ . Logic:  $v_j$  belongs to exactly those spatial tetrahedra that do not skip  $v_j$ ;  $v_4$  is in all three, giving full  $S_3$ .

*T-covariance.*  $R: v_j \leftrightarrow v_{4-j}$ ,  $R^2 = I$ ,  $RDR = D$  (exactly),  $RD^{(j)}R = D^{(4-j)}$ . The full Dirac is  $T$ -invariant; past and future are  $T$ -conjugate.

*$S_3$  breaking at finite  $k$ .*  $D(k)$  does not commute with  $S_3$  at  $k \neq 0$ ; at  $k = 0$  it commutes exactly. The breaking arises from inter-cell Bloch phases—a physical finite-momentum anisotropy.

A selection rule emerges that will prove decisive for the mass hierarchy. In the  $S_3$ -adapted generation basis  $(e_s, e_1, e_2)$ , the zero-momentum Dirac matrix satisfies  $D_0(e_s, e_2) = 0$  exactly, while  $D_0(e_s, e_1) = -4.08 \neq 0$ . Generation 2 is exactly decoupled from the singlet at  $k = 0$ . At finite  $k$ :  $|A(e_s, e_1)|/|A(e_s, e_2)| \approx 12$ —a robust 2+1 channel structure. This is the algebraic origin of the electron's lightness.

## 19.8 Boundary transfer kernel

When the sliding window advances by one tick, what survives?

The shared  $K_4$  boundary carries 6 edge phases, 3 of which are gauge. The remaining 3 are physical holonomy modes. The Hodge Laplacian on  $K_4$  edges is  $\Delta_1 = 4I$ —all three physical modes have the same stiffness. Local filling is exactly  $S_3$ -symmetric: no generation is preferred within a single window.

The inter-window transfer can be written as an explicit finite quadratic kernel. Let  $C_{\text{in}}$  and  $C_{\text{out}}$  denote the maps from the  $K_5$  edge phases to the physical face-holonomies of the past and future  $K_4$  boundaries. For a prescribed past-boundary holonomy  $h$ , define the one-cell filling

$$x_*(h) = \arg \min_x \|Bx\|^2 \quad \text{subject to} \quad C_{\text{in}}x = h. \quad (26)$$

The transfer operator is then

$$T_{K_4}h = P_{\text{out}}C_{\text{out}}x_*(h), \quad (27)$$

where  $P_{\text{out}}$  projects onto the rank-three physical holonomy subspace of the future  $K_4$ . The reproducibility script constructs this  $3 \times 3$  matrix directly and computes

$$\sigma(T_{K_4}) = \{1, \frac{1}{4}, \frac{1}{4}\}. \quad (28)$$

The protected singular vector is the physical projection of the common colour-face holonomy  $\Phi_{123}$ —the phase accumulated around the triangle of colour vertices  $\{v_1, v_2, v_3\}$ . Its three edges are shared between the past and future  $K_4$  boundaries in a one-step slide, so this component is not integrated out by the filling. The transfer is therefore lossless for this projected colour-face direction.

The two suppressed modes are orthogonal to the projected colour-face direction and live in the spatial holonomy complement. Their singular value is  $1/4$ , the inverse of the nonzero  $K_4$  boundary stiffness eigenvalue  $4 = |V(K_4)|$ . The ratio between protected and suppressed transfer is therefore the topological boundary factor 4.

The pattern  $\{1, 1/4, 1/4\}$  is reproduced at every one-step slide because every slide contains the same  $K_4$  filling problem. It is not inserted as a phenomenological damping rule; it is the singular spectrum of the finite  $K_4$  boundary transfer kernel.

*One-step filling.* Adding  $v_4$  to the past boundary  $K_4 = \{v_0, v_1, v_2, v_3\}$  completes  $K_5$ . Gaussian integration over 4 new edges gives effective boundary action  $M_{\text{eff}} = P_{\perp}$  with eigenvalues  $\{0, 0, 0, 1, 1, 1\}$ . Total kernel:  $\{0, 0, 0, 5, 5, 5\}$ —all three physical holonomies receive the same scale  $\sqrt{5}$ .

*Finite lifetime of the colour face.* The colour face of window  $n$  is  $\Phi_{n+1, n+2, n+3}$ , supported on the three edges of the triangle  $(n+1, n+2, n+3)$ . They survive in window  $n+1$ , but in window  $n+2$  vertex  $n+1$  leaves  $K_4$ , killing two of three edges. Lifetime = 2 windows. Successive colour

faces share only 1 of 3 edges; after 3 steps the set is completely refreshed.  $\sigma = 1$  is a one-step lossless transfer, not infinite protection.

*Origin of the number 4.*  $B^\top B = \text{graph Laplacian of } K_4$ . Its nonzero eigenvalue is  $4 = |V(K_4)|$ . The ratio  $\sigma_1/\sigma_2 = 4$  is a topological invariant: the boundary mass equals the number of boundary vertices.

*Physical interpretation.* The boundary  $K_4$  carries 3 holonomies. Local filling does not distinguish them (eigenvalue 5). Inter-window transfer does: the all-colour  $\Phi_{123}$  is lossless, the spatial holonomies are suppressed by 4. This is the boundary realisation of the 1+2 pattern found independently in the Dirac sector (§19.7).

## 19.9 The photon

A single  $K_5$  cell has no massless particle. Its physical Hessian  $H = B^\top B$  has spectrum  $\{0^4, 5^6\}$ : four gauge zeros, six gapped modes. The photon is not a single-cell object; it is a collective mode of the synchronized network.

*Source.* The ordered vacuum has  $\Phi_f \simeq 0$ . The photon is not a mandatory nonzero background fluctuation. It is a response mode of the compact edge-phase ensemble around the ordered background. A small edge-phase perturbation changes triangular holonomies; the Wilson action then propagates the compensating phase response through neighbouring seams. Gauge fixing removes vertex-local redundancy, but physical holonomy response modes remain.

*Propagation.* Across one  $K_4$  seam the protected U(1) seam branch has singular value  $\sigma = 1$ . This does not mean that the same triangular colour face remains literally present through arbitrarily many windows. The colour face is refreshed by sliding. The invariant statement is that each step contains a protected U(1) seam subspace and the transfer from one protected subspace to the next is lossless. The photon is the coherent composition of these refreshed protected transfers.

Subleading seam branches have  $\sigma_i < 1$ , for example  $\sigma_i \simeq 1/4$ , and therefore decay exponentially across repeated seams. Only the renewed protected U(1) branch has spectral radius one in the long-distance transfer.

*Masslessness.* The Wilson action depends on the holonomy  $\Phi_f$ , which is the discrete curl of the edge-phase field. A mass term  $m^2\theta_e^2$  depends on the phase itself, which is gauge-dependent and therefore forbidden. Only the curl (holonomy) is physical. In the long-distance limit, the surviving U(1) response has the form  $H_{ab}(q) = \kappa(q^2\delta_{ab} - q_a q_b) + O(q^4)$ , where the longitudinal component is pure gauge. The two transverse modes have dispersion  $\varepsilon = \sqrt{\kappa}|q|$  at small  $q$ : linear, gapless, with speed  $v = \sqrt{\kappa} = c$  in lattice units. The response kernel has a massless pole  $H^+(q) \sim P_T/q^2$ .

*Uniqueness.* One colour face per  $K_4$  seam  $\rightarrow$  one  $\sigma = 1$  branch  $\rightarrow$  one massless species. Three spatial channels minus one longitudinal gauge mode  $\rightarrow$  two transverse polarizations. One photon, spin 1, two helicities.

*Arithmetic analogue.* Before the same compact phase is placed on a causal DAG, the identity twist of the cover monoid has response

$$L(s, \chi_{\text{triv}}) = \zeta(s).$$

The companion note calls this the arithmetic photon. It is not the physical photon and does not propagate in spacetime; it occupies the same structural slot in the Layer-0 branch: the untwisted U(1) response. In the generated-complete  $K_5$  hierarchy, RH says that this completed arithmetic photon has no phantom inner phase channel; equivalently, all its zero readouts are critical-line nodes.

*Coupling.* A charged Type-B defect modifies the same edge phases through which the photon propagates. Their interaction is automatic: both live on the same U(1) edge-phase layer. Coupling strength:  $\alpha_{\text{bare}}^{-1} = |A_5| = 60$  (§16.1). The low-energy value  $\alpha_{\text{em}}^{-1} \simeq 137$  is the dressed

channel response of §16.3: lepton channels plus open charged colour channels, with NLO vector-threshold completion still separated from the theorem-level bare coupling.

## 19.10 IR structure and zero-mode residues

The Dirac selection rule  $D_0(e_s, e_2) = 0$  has a gauge-sector counterpart. The infrared mode of the temporal chain, projected onto the colour basis, gives  $\langle \text{IR}, e_s \rangle = 6.93$ ,  $\langle \text{IR}, e_1 \rangle = \sqrt{2}$ , and  $\langle \text{IR}, e_2 \rangle = 0$ —exactly, from the identity  $5 - 2 \cdot 4 + 3 = 0$ . Channel  $e_2$  has zero overlap with the massless branch. Its fluctuations converge to a finite limit as the chain grows:  $\langle t_2^2 \rangle \rightarrow 0.155/\beta$  for  $L \rightarrow \infty$ .

**Theorem 19.2** (Zero-mode residues). *On the temporal  $K_5$ -chain, the Bloch propagator restricted to the generation-edge basis has residues*

$$Z_{ss} = 2/5, \quad Z_{11} = 1/15, \quad Z_{e_2 e_2} = k^2/180. \quad (29)$$

*All three are exact rationals.*

The ratio  $Z_{11}/Z_{ss} = 1/6$  is  $k$ -independent. The  $e_2$  residue vanishes at  $k = 0$  and grows only quadratically—not an exponential gap but a quadratic screening, with the full analytic form  $G_{e_2 e_2}^0(k, z) = zk^2/(180(30k^2 - z^2))$ .

This is the mechanism that makes the electron light. It is not the “smallest Yukawa coupling to a heavy scalar.” It is a near-complete absence of access to the massless branch—a qualitatively different explanation. The heaviest channel is the most decoupled from the massless sector.

*Gap estimate.* With the scaling  $D \times G$ :  $\text{gap}(e_2) = 0.289 \times \langle t_2^2 \rangle$ . At  $\beta = |A_5|/(4\pi) \approx 4.77$ :  $\text{gap}(e_2) \approx 9.4 \times 10^{-3}$  in lattice units—a self-consistent one-step estimate.

*Three manifestations of  $e_2$  selection.* The same  $e_2$ -decoupling appears in three independent calculations: (i) Dirac selection rule  $D_0(e_s, e_2) = 0$  (§19.7); (ii) protected mode  $\Phi_{123}$  (§19.8); (iii) IR covariance  $\langle \text{IR}, e_2 \rangle = 0$  (this section). The first and third are exact zeros from the same identity ( $5 - 2 \cdot 4 + 3 = 0$ ). The second is consistent; the holonomy  $\rightarrow$ flavour intertwiner requires further clarification.

## 19.11 Fermion transfer and splitting architecture

The 31-dimensional cochain space splits canonically as  $\Omega = S \oplus P \oplus F \oplus I$  (shared memory, past-only, future-only, interior:  $7+8+8+8$ ), and the transfer operator is exactly  $S_3$ -equivariant.

Restricted to the flavour triplet—past generation edges  $(0, 1), (0, 2), (0, 3)$  and their future counterparts  $(1, 4), (2, 4), (3, 4)$ —the transfer takes a striking form:

$$T_{\text{flav}}(z) = \frac{-1}{4 - z^2} \times I_3. \quad (30)$$

This is the resolvent of the Hodge Laplacian on  $K_4$  with a pole at  $z = \pm 2$ . All three generations have the same boundary mass. A single window does not split flavours.

Where does the splitting come from? Not from a single cell but from the chain. Three layers contribute, each breaking less symmetry than the one before:

At the level of one window,  $S_3$  is exact. At  $k = 0$  on the chain, the Dirac selection rule  $D_0(e_s, e_2) = 0$  creates a 2+1 pattern. At  $k \neq 0$ , the inter-window holonomy  $\sigma = \{1, 1/4, 1/4\}$  and the IR screening complete the hierarchy.

The bridge formula  $Z_{11}/Z_{22}(k_{\min}) = 3L^2/\pi^2$  grows quadratically with chain length and matches the observed  $m_\mu/m_e = 207$  at  $L = 26$ . The structural constant  $Z_{ss}/Z_{11} = 6$  is independent of  $L$ .

*Two mass scales from  $K_5$ .* Boundary:  $m_\partial = \sqrt{4} = 2$  (Hodge  $K_4$ ). Bulk:  $m_{\text{bulk}} = \sqrt{5}$  (Hodge  $K_5$ ). Ratio:  $m_\partial/m_{\text{bulk}} = 2/\sqrt{5} = \sqrt{4/5}$ —a number from  $K_5$ , not a parameter. Both are  $S_3$ -exact. Splitting comes not from one window but from the temporal chain.

*Free pole basis.* Diagonalisation of the  $3 \times 3$  matrix  $G^{\text{rad}}(z)$  singles out three eigen-operators with distinct first poles:

Eigen-operator	First pole $\lambda^2$	Composition
$u_{\sqrt{5}}$	5	$\approx -0.378 e_s + 0.926 e_1$
$u_{2\sqrt{2}}$	8	$= e_2$ (exactly)
$u_3$	9	$\approx 0.775 e_s + 0.632 e_1$

In channel language:  $m(e_s) = m(e_1) = \sqrt{5}$ ,  $m(e_2) = 2\sqrt{2}$ . In eigen-operator language:  $\sqrt{5}$ ,  $2\sqrt{2}$ , 3. No contradiction—different bases.

With dressed IR mass:  $Z_{11}/Z_{22}^{\text{eff}} = 12/[(2\pi/L)^2 + m_{\text{IR}}^2/30]$ , saturating at  $L \gg \xi = \sqrt{30}/m_{\text{IR}}$  to  $(Z_{11}/Z_{22}^{\text{eff}})_{\text{sat}} = 360/m_{\text{IR}}^2$ . The bare 1D hierarchy grows as  $L^2$  until generating a self-consistent  $m_{\text{IR}}$ , then saturates.

## 19.12 Charged lepton masses from $K_5$ causality

The Standard Model has two large mass ratios among charged leptons:  $m_\tau/m_\mu \approx 17$  and  $m_\mu/m_e \approx 207$ . Their very different magnitudes suggest that no single mechanism produces both.  $K_5$  confirms this: the two ratios arise on different causal scales.

### 19.12.1 Two causal barriers

Mechanism	Computes	Physical meaning
Local one-window curvature cost	$W_s = 13/3$ , $W_1 = 1/4$ , $W_2 = 17/12$	curvature volume seen by the channel within one window
Chain IR screening along the stream	$Z_{ss} = 2/5$ , $Z_{11} = 1/15$ , $Z_{22} = k^2/180$	how strongly the channel couples to the soft branch

### 19.12.2 Carrier identity

$W$  and  $Z$  act not on two similar objects but on the same flavour carrier: the  $3 \times 3$  generation-edge space  $\mathbb{R}^3 = \{e_s, e_1, e_2\}$ —Triplet IV of §18.4. This turns the factorisation from a heuristic into a causally natural candidate.

### 19.12.3 Causal factorisation

$[M_W, M_Z(L=26)]$  relative norm = 0.125; MC correlation  $r = -0.16$ . The action splits as  $S = S^{\text{loc}} + S^{\text{IR}}$  at leading order; the cross-term is  $O(1/\beta)$ .

### 19.12.4 Leading causal mass law

The first ratio is local—set within a single  $K_5$  window by the channel-resolved gauge curvature cost. The curvature weights  $W_s = 13/3$ ,  $W_1 = 1/4$ ,  $W_2 = 17/12$  give

$$m_\tau/m_\mu = W_s/W_1 = \frac{13/3}{1/4} = \frac{52}{3} = 17.33 \quad (\text{observed: } 16.82, 3\%). \quad (31)$$

Note the operational status:  $52/3$  is the exact local curvature barrier (a structural leading-order component). The final physical pole ratio (observed: 16.82) requires the full chain/dressing corrections, making the 3% deviation a measure of NLO dressing rather than a failure of the exact local ratio.

The second is global—set by IR screening along the full causal depth  $L^* = 26$ :

$$m_\mu/m_e = 3L^{*2}/\pi^2 = 205.5 \quad (\text{observed: } 206.8, 0.6\%). \quad (32)$$

Ratio	$K_5$	Experiment	Agreement
$m_\tau/m_\mu$	$52/3 = 17.33$	16.82	97%
$m_\mu/m_e$	$3 \times 26^2/\pi^2 = 205.5$	206.8	99.4%
$m_\tau/m_e$	$52 \times 26^2/\pi^2 = 3562$	3477	97.6%

### 19.12.5 Structural constant 6—double derivation

$6 = Z_{ss}/Z_{11}$  (kinematics, residue ratio) =  $\exp(0.475 \times (1/W_1 - 1/W_s))$  (dynamics, curvature). The number 6 appears twice—from chain kinematics and from one-window dynamics—two layers describing the same physics from different sides.

*Matrix upgrade.* A naive matrix hybrid (combining both mechanisms into a single matrix diagonalisation) fails: rank collapse mixes the two causally separated scales into one, pulling both ratios toward the same order. The diagonal law is the correct LO: two causally separated scales, one local (curvature) and one global (chain screening). Matrix corrections are NLO ( $\sim 12\%$ ).

*Non-perturbative benchmarks.* Wilson MC at  $L = 26$ ,  $\beta = 15/\pi$ :  $C_{ss}/C_{11} = 6.06 \pm 0.01$  (structural constant 6 confirmed);  $C_{11}/C_{22} = 116 \pm 1$  vs SM target 207—gap  $\sim 1.8\times$ , traceable to five sources (RHMC dressing, LCP, NLO defect, flavour map, 4D dynamics).

### 19.12.6 Absolute scale and relative hierarchy separate

Type B defect sets the absolute ledger of a charged lepton, while the projection–gravity fixed point converts the measured electron pole mass into the gravitational Planck scale:

$$M_{\text{Pl}} = m_e \exp\left(51 + \frac{19}{36}\right).$$

Curvature + screening then set the relative distribution among  $\tau, \mu, e$ . These are two mechanisms with different causal origins: first anchor the absolute scale with the minimal charged defect and the projection–gravity stiffness correction, then determine how easily a given flavour channel passes along the stream.

Absolute masses: with  $m_e$  as the dimensional unit,  $M_{\text{Pl}}$  is derived at the  $\sim 10^{-4}$  level, and the charged-lepton ratios give  $m_\mu \approx 107$  MeV (99%) and  $m_\tau \approx 1853$  MeV (96%).

## 19.13 Causal depth $L^* = 26$ (theorem)

**Theorem 19.3** (Causal depth).  $L^* = \dim \Omega^{\geq 1}(K_5) = 10 + 10 + 5 + 1 = 26$  (*exact combinatorial invariant*).

Structural decomposition:  $\dim \Omega(K_5) = 31 = 26 + 4 + 1$  (relational + gauge + constant). Dynamical shadow:  $L_{\text{dyn}} = \beta\sqrt{30} \approx 26.15$ . Resonant realisation:  $k_{\text{IR}} \cdot L^* = 2\pi$ .

Three independent derivations converge to  $L^* = 26$ : (1) Combinatorial:  $\dim \Omega^{\geq 1}(K_5) = 31 - 5 = 26$ . (2) Dynamical:  $\beta\sqrt{30} \approx 26.15$ . (3) Resonant:  $k_{\text{IR}} \cdot L^* = 2\pi$ .

$L^* = 26$  is not the size of the world but the reading depth of the world by a single causal stream.

## 19.14 The 30 self-matching identity

The recurring number 30 is not used below as a loose numerological echo. It is the same matching invariant of the  $K_5$  window seen through several layer projections. We name it

$$\mathcal{N}_{30}.$$

**Definition 19.4** ( $K_5$  thirtyfold matching invariant). In the normalisations used throughout this manuscript,

$$\mathcal{N}_{30} := \text{Tr}'(d_2^\top d_2) = \text{Tr}(H_{\text{vac}}|_{\text{phys}}) = \sum_{d=1}^4 d^2 = \frac{|S_5|}{4} = 2 \dim(\bar{\mathbf{5}} \oplus \mathbf{10}) = 30. \quad (33)$$

Here  $d_2$  is the face-to-edge boundary operator of the 4-simplex. The equality

$$\text{Tr}'(d_2^\top d_2) = 6 \cdot 5 = 30$$

uses the six physical two-form modes, each with stiffness 5; equivalently, each of the ten edges of  $K_5$  lies in exactly three triangular faces, so  $|E(K_5)| |F_{\text{per edge}}| = 10 \cdot 3 = 30$ . The oriented half is

$$\frac{|A_5|}{4} = 15 = \dim(\bar{\mathbf{5}} \oplus \mathbf{10}).$$

The same oriented count also supplies the arithmetic conductor

$$q_{K_5} = |A_5| = 60, \quad \varphi(60) = 16 = 1 + 15.$$

The companion theorem sharpened in §18.7 shows that this is unique among alternating windows:

$$\varphi(|A_n|) = 2^{n-1} \iff n = 5.$$

Equivalently,  $\mathcal{N}_{30}$  is the vacuum physical stiffness trace, the Dirac velocity squared  $v_D^2 = 30$ , the square-conductance of the four allowed causal step lengths, the unoriented step-normalised symmetry factor, and the Dirac-doubled minimal carrier size. The dynamical shadow is then  $L_{\text{dyn}} = \beta \sqrt{\mathcal{N}_{30}} \approx 26.15$ .

The oriented window group has order  $|A_5| = 60$ , while the full unoriented window symmetry has order  $|S_5| = 120$ . A sliding  $K_5$  step introduces four causal step directions relative to the previous  $K_4$  seam. The same invariant is therefore also the conductance of the four allowed causal step lengths,

$$1^2 + 2^2 + 3^2 + 4^2 = \mathcal{N}_{30} = 30.$$

Thus 30 is simultaneously the unoriented step-normalised symmetry factor, the Dirac-doubled minimal carrier size, and the canonical conductance of a sliding  $K_5$  winding layer.

More generally, for a complete graph  $K_n$  define

$$C_n = \sum_{d=1}^{n-1} d^2 = \frac{n(n-1)(2n-1)}{6},$$

$$D_n = 2 \left( n + \binom{n}{2} \right) = n(n+1),$$

and

$$R_n = \frac{|S_n|}{n-1} = \frac{n!}{n-1}.$$

Then  $C_5 = D_5 = R_5 = 30$ . The equation  $C_n = D_n$  has the positive integral solution  $n = 5$  only, since

$$\frac{n(n-1)(2n-1)}{6} = n(n+1) \iff 2n^2 - 9n - 5 = 0.$$

The equation  $R_n = D_n$  reduces, for  $n > 2$ , to

$$(n-2)! = n+1,$$

whose complete-graph solution is again  $n = 5$ . In this precise sense  $K_5$  is the unique complete-graph self-matching point between step-normalised symmetry, Dirac-doubled carrier size, and causal-step conductance.

The companion arithmetic note develops the associated periodic sliding-chain winding theorem. The clean winding channel first appears at

$$L_{\min} = 2|V(K_5)| - 1 = 9.$$

For  $L \geq 9$  the periodic chain has  $H_1 \cong \mathbb{Z}$  and no higher homology; for  $6 \leq L \leq 8$  the overfilled complexes carry parasitic higher cycles (while  $L = 5$  is contractible), with the resolution cascade

$$\beta_4(L = 6) \rightarrow \beta_3(L = 7) \rightarrow \beta_2(L = 8) \rightarrow \beta_1(L = 9).$$

The intermediate  $L = 8$  value  $\beta_2 = 3$  is not the  $Z_3$  spatial/generation triplet; the  $Z_8$  translation action on  $H_2$  has characters  $\{\omega^2, \omega^4, \omega^6\}$ . At the threshold the fresh-simplex row  $c_d = \binom{4}{d} = (1, 4, 6, 4, 1)$  gives  $f_d(X_L) = c_d L$  for all  $L \geq 9$ .

With the canonical equal-conductance edge-current metric, the unit harmonic winding has components  $w_{j,d} = d/30$  for  $d = 1, 2, 3, 4$  and norm  $\|w_{\text{harm}}\|^2 = L/30$ . Together with  $\beta = 15/\pi$ , this yields the one-winding Villain action

$$S_N = \frac{\pi L}{900} N^2.$$

The corresponding self-dual topological length is  $L_{\text{sd}} = 900 = 30^2$ . This proves the  $K_5$  one-winding theta normalisation for the clean winding regime  $L \geq 9$ ; it is not a proof of the Riemann hypothesis and it is not by itself a scale dictionary to  $M_{\text{Pl}}$  or  $V_0$ .

The companion note also proves the finite conductor-60 twist-packet identity

$$(\widehat{\mathbb{Z}/60\mathbb{Z}})^\times = 1_\gamma \oplus \bar{\mathbf{5}} \oplus \mathbf{10}.$$

Thus 30 and 60 play complementary roles: 30 is the Dirac-doubled carrier/conductance matching invariant, while  $60 = |A_5|$  is the oriented conductor whose finite arithmetic shell has  $\varphi(60) = 16 = 1 + 15$  sectors. This is a structural sector matching, not a proof of RH or GRH.

### 19.15 Koide [7] and $\mathbb{Z}_3$

$Q = 2/3$  (experiment: 0.666656, deviation 0.002%).  $M_K^2 = v_H \exp(-60/9) = 313.1$  MeV ( $\Delta = 0.2\%$ ). All three masses to 0.3% from  $\alpha^{-1} = 60$ ,  $N_{\text{gen}} = 3$ ,  $+v_H$ .

### 19.16 EW bilinear kernel (theorem)

The EW bilinear carrier is  $\mathcal{B}_{\text{EW}} = 3_L \otimes 3_R$  (dimension 9), where  $3 =$  generation triplet (Triplet IV). Under  $A_4$ :  $3 \otimes 3 = 1 \oplus 1' \oplus 1'' \oplus 3_a \oplus 3_b$ .

**Theorem 19.5.** *The one-step  $K_5$  kernel on  $\mathcal{B}_{\text{EW}}$  with Haar averaging over  $A_5$ :*

$$T_{\text{EW}} = \frac{3}{5} P_1 + \frac{3}{10} P_{II} + 0 \cdot P_{\text{rest}}. \quad (34)$$

*Proof.*  $T_{\text{EW}}[(k, l), (i, j)] = (1/|A_5|) \sum_{g \in A_5} [g(i) = k] \cdot [g(j) = l] \cdot [k, l \in \text{FLAV}]$ . For the trivial singlet  $|1\rangle = (E_{11} + E_{22} + E_{33})/\sqrt{3}$ : for fixed  $(i, k) \in \text{FLAV}^2$ , the number of  $g \in A_5$  with  $g(i) = k$  is  $|A_5|/|V(K_5)| = 60/5 = 12$ . Hence  $T_{\text{EW}}[(i, i), (k, k)] = 12/60 = 1/5$ . Summing over 9 pairs and normalising by  $1/3$ :  $\langle 1|T_{\text{EW}}|1\rangle = \frac{1}{3} \cdot 9 \cdot \frac{1}{5} = 3/5$ .  $\square$

The leading eigenvalue:

$$\lambda_1 = \frac{N_{\text{gen}}}{|V(K_5)|} = \frac{3}{5}. \quad (35)$$

An exact  $K_5$  rational—no free parameters, one number from combinatorics.

After  $L^*/2 = 13$  steps:  $y_{\text{EW}} = (3/5)^{13} \approx 1.55 \times 10^{-3}$ .

Comparison:  $(3/5)^{13}$  agrees with the  $K_5$ -derived  $y_K$  (from §19.12 + Koide) to 0.6%;  $\exp(-60/9)$  from §19.15 agrees with the experimental  $y_K$  to 0.2%. The two formulae describe different levels: discrete kernel-level vs continuum/resummed envelope. The  $(3/5)^{13}$  vs  $\exp(-60/9)$  gap ( $\sim 2\%$ ) reflects the 2% errors of the §19.12 mass chain (bare vs dressed relation), not a  $K_5$  puzzle.

### 19.17 EW scale architecture: the leptonic bridge

In  $K_5$ , the EW scale  $v = 246$  GeV has no independent dimensional anchor. It is derived entirely from the leptonic sector via the Koide bridge:

$$v = M_K \cdot \exp\left(\frac{|A_5|}{N_{\text{gen}}^2}\right) = M_K \cdot \exp\left(\frac{60}{9}\right), \quad (36)$$

where  $M_K = (\sum_f \sqrt{m_f}/3)^2$  is the Koide mean computed from  $K_5$ -derived lepton masses. Numerically:  $v = 251$  GeV (2% off experimental 246.22 GeV).

The full causal chain of the EW sector:

$$m_e \xrightarrow{51+19/36} M_{\text{Pl}} \xrightarrow{3L^{*2}/\pi^2, 52/3} (m_\mu, m_\tau) \xrightarrow{\text{Koide}} M_K \xrightarrow{\exp(60/9)} v \xrightarrow{3/\sqrt{35}} m_H. \quad (37)$$

Key observation:  $W$  and  $H$  are not independent. In the SM,  $v$  is a free parameter (one of 19). In  $K_5$ ,  $v$  is a derived ratio from the leptons. There is no separate “anchor for the EW sector”. The sole dimensional unit choice may be taken to be  $m_e$ ; the Planck scale is then reconstructed by the projection–gravity fixed point of §19.5.

	SM	$K_5$
$v_{\text{EW}}$	free parameter	derived via Koide bridge
$m_H$	$y_H \times v$ ( $y_H$ free)	$v \times 3/\sqrt{35}$ (§26.5)
$M_W$	$g_2 \times v/2$	$g_2$ derived from $\alpha_{K_5}$ and $\sin^2 \theta_W$
Yukawas $y_f$	9 free	derived via causal chain
Hierarchy problem	unresolved	derived from leptons

Status: all EW scales ( $v$ ,  $M_W$ ,  $M_Z$ ,  $m_H$ ) derive from one dimensional anchor  $m_e$ , with  $M_{\text{Pl}}$  generated internally by the absolute-scale bridge of §19.5. The hierarchy problem is formally resolved through the leptonic chain.

### 19.18 $K_4$ matching structure and Yukawa

After causal fixing of vertex  $v_0$ , the residual graph  $K_4$  (vertices  $\{1, 2, 3, 4\}$ ) has exactly 3 perfect matchings ( $K_{2n}$  has  $(2n-1)!!$ ; for  $n = 2$ :  $3!! = 3$ ):  $M_1 = \{(1, 2), (3, 4)\}$ ,  $M_2 = \{(1, 3), (2, 4)\}$ ,  $M_3 = \{(1, 4), (2, 3)\}$ . The 6 edges of  $K_4$  partition into 3 matchings of 2 edges each; the partition is unique up to relabelling.

$S_4$  on vertices permutes the matchings; the stabiliser of all three is  $V_4 \cong \mathbb{Z}_2 \times \mathbb{Z}_2$ . The induced quotient  $S_4/V_4 \cong S_3$ ; for even permutations  $A_4/V_4 \cong \mathbb{Z}_3$ —cyclic permutation  $M_1 \rightarrow M_2 \rightarrow M_3$ . Each matching carries  $\mathbb{Z}_3$  charge  $\omega^k$ .

Tree-level Yukawa in the vertex basis:  $Y = \sum T(b, a) \cdot T(a, b) = (3/2) \cdot I$ —three generations exactly degenerate. At finite gauge field:  $|Y_1| : |Y_2| : |Y_3| = 2.8 : 1 : 2.8$  at  $\beta = 5$ . Generation 2 is the weakest at all  $\beta$ , consistent with the Dirac selection rule  $D_0(e_s, e_2) = 0$  (§19.7).

### 19.19 CKM hierarchy from $A_4$ [32]

Three generations = triplet  $\mathbf{3}$  of  $A_4$ . With exact  $A_4$ :  $Y_u$  and  $Y_d$  are simultaneously diagonalisable  $\rightarrow V_{\text{CKM}} = I$  (no mixing). CKM mixing requires  $A_4$  breaking.

The key mechanism: the up-sector preserves  $\mathbb{Z}_3 \subset A_4$ , the down-sector preserves  $\mathbb{Z}_2 \subset A_4$ . The misalignment of residual symmetries generates CKM.

With perturbative breaking parameter  $\varepsilon \ll 1$ :  $V_{us} \sim \varepsilon$ ,  $V_{cb} \sim \varepsilon^2$ ,  $V_{ub} \sim \varepsilon^3$ —the Wolfenstein hierarchy  $\lambda : \lambda^2 : \lambda^3$ .

Power-counting check:  $\log |V_{cb}| / \log |V_{us}| = 2.14$  (predicted: 2, 7% off). Cabibbo hint:  $\lambda_{\text{Cabibbo}} = \sin(2/9) = 0.220$  (experiment: 0.225,  $-2.2\%$ ). A single  $\mathbb{Z}_3$  phase appears to control both lepton masses and CKM.

Structural predictions: power-law  $\varepsilon : \varepsilon^2 : \varepsilon^3$ ; 3 mixing angles from  $N_{\text{gen}} = 3$ ; one CP phase from  $(3-1)(3-2)/2 = 1$ ; approximate  $\mu$ - $\tau$  symmetry from  $\mathbb{Z}_2 \subset A_4$ . Quantitative: exact  $\theta_C \approx 13^\circ$  and  $\delta_{CP} \approx 68^\circ$  require the flavon sector (not fixed by minimal  $K_5$ ).

### 19.20 Neutrino sum rule

The  $A_4$  structural breaking  $A_4 \rightarrow \mathbb{Z}_3$  preserves the tribimaximal first column,

$$|U_{e1}|^2 = \cos^2 \theta_{12} \cos^2 \theta_{13} = \frac{2}{3}.$$

Using the measured value  $\sin^2 \theta_{13} \simeq 0.02215$ , the minimal  $K_5$  relation fixes

$$\sin^2 \theta_{12}^{K_5} = 1 - \frac{2/3}{\cos^2 \theta_{13}} \simeq 0.3182.$$

The first 59.1-day JUNO result gives [39]

$$\sin^2 \theta_{12} = 0.3092 \pm 0.0087, \quad \Delta m_{21}^2 = (7.50 \pm 0.12) \times 10^{-5} \text{ eV}^2.$$

The  $K_5$  prediction is therefore

$$\frac{0.3182 - 0.3092}{0.0087} \simeq 1.04\sigma$$

above the JUNO central value. Thus the first JUNO data do not falsify the minimal  $K_5$  relation; the prediction remains viable within standard statistical margins. For comparison, the NuFIT 6.0 normal-ordering baseline gives  $\sin^2 \theta_{12} = 0.307_{-0.011}^{+0.012}$  [38], so the JUNO central value is not moving away from the  $K_5$  value.

Future JUNO exposure is expected to compress the uncertainty to the  $\sim 0.003$  level. If the central value remains near 0.309, the minimal  $K_5$  model will face severe  $> 3\sigma$  tension. If the central value shifts or stabilizes toward 0.315–0.318, this relation will become a decisive confirmation. This remains the sharpest near-term experimental falsifier of minimal  $K_5$ .

### 19.21 FCNC and charge quantisation

No tree-level FCNC: one Higgs doublet (T7) + three generations  $\rightarrow$  GIM mechanism. The  $Z$ -boson coupling through  $T_3$  and  $Y$  is diagonal in flavour space; CKM appears only in charged currents ( $W^\pm$ ). FCNC appear only in loops, suppressed as  $(m_{\text{heavy}}^2/M_W^2) \times (\alpha/\pi)$ .

Charge quantisation: in the standard formulation of the SM (prior to imposing anomaly cancellation), hypercharges are free parameters. In  $K_5$ :  $Y \in \mathbb{Z}/6$  is an immediate structural consequence of  $|\text{Weyl}(\text{SU}(3))| = 6$ .  $Q = T_3 + Y \in \mathbb{Z}/6$ . Atomic neutrality  $\sum Q_i = 0$  follows from anomaly cancellation, not from SU(5) GUT.

## 19.22 Anomalous magnetic moment

Loop corrections in  $K_5$  are not imported from QED—they arise as connected cumulants of the compact Wilson action. The full derivation of loop structure, the Schwinger term  $a_e = \alpha/(2\pi)$ , and the  $K_5$ -native origin of fermion loops is given in §24.7.

## 19.23 Antimatter and CPT

In  $K_5$ , antimatter is not introduced as a second species of object. It is the orientation-reversed form of the same causal motif. A charged Type B defect carries an oriented U(1) edge transporter. Reversing the orientation of the defect sends

$$g_e = e^{i\theta_e} \mapsto g_e^{-1} = e^{-i\theta_e},$$

and therefore reverses the charge. The positron is the orientation-reversed Type B defect corresponding to the electron.

The same logic applies to the discrete spacetime operations. Charge conjugation  $C$  reverses the U(1)-orientation of the defect. Parity  $P$  acts by exchanging the spatial channels of the  $K_5$  window. Time reversal  $T$  is not an allowed dynamical evolution of the DAG, but it is a well-defined orientation reversal of the boundary kernel. The combined operation  $CPT$  returns the unoriented phase-closure data of a stable motif to itself.

Thus  $K_5$  does not require antimatter to be a separate ontology. Matter and antimatter are opposite orientations of the same causal motif.

## 20 Prohibitions

### 20.1 $\bar{\theta} = 0$ from the orientational symmetry of $K_5$

**The problem.** The SM allows the CP-violating term  $\bar{\theta} \cdot (g^2/32\pi^2) \text{Tr}(F\tilde{F})$ . Experiment requires  $|\bar{\theta}| < 10^{-10}$ , but the SM does not explain this smallness—one of the oldest open problems.

**Resolution from  $K_5$ .** The topological charge  $Q$  is a quadratic pseudoscalar transforming in the sign representation of  $S_5$ . The gauge action and measure are invariant under the full  $S_5$ , including odd permutations. Therefore  $\langle Q \rangle = 0$  exactly.

**Explicit construction.**

1. Face holonomies  $\{\Phi_f\}_{f=1}^{10}$  form the 10-dimensional representation  $V$  of  $S_5$ .
2.  $\text{mult}(\text{sign}, V) = 0$ —no linear pseudoscalar exists.
3.  $\text{mult}(\text{sign}, \text{Sym}^2 V) = 1$ —a unique quadratic pseudoscalar:

$$Q = \sum_{f_1, f_2} C_{f_1 f_2} \Phi_{f_1} \Phi_{f_2}, \quad (38)$$

where  $C_{f_1 f_2} \neq 0$  only for face pairs sharing exactly one vertex and covering all 5 vertices of  $K_5$ . The sign of  $C$  is determined by the Levi-Civita orientation of the 4-simplex—the lattice analogue of  $\varepsilon_{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma}$ .

4. Verified:  $Q(\sigma \cdot \theta) = \text{sign}(\sigma) \cdot Q(\theta)$  for all 120 elements of  $S_5$  on multiple random configurations (ratio =  $\pm 1.0000$  without exception).
5. **Symmetry argument.**  $S = \beta \sum (1 - \cos \Phi)$  is  $S_5$ -invariant ( $\cos$  is even). The measure  $d\mu = e^{-S} \prod d\theta$  is  $S_5$ -invariant. For any odd  $\tau$ :  $\langle Q \rangle = \langle Q(\tau \cdot \theta) \rangle = \text{sign}(\tau) \langle Q \rangle = -\langle Q \rangle \implies \langle Q \rangle = 0$  exactly.
6. Any term in  $S_{\text{eff}}$  proportional to  $Q$  is forbidden by  $S_5$ : its coefficient = 0. Therefore  $\bar{\theta} = 0$ .

**How this differs from standard approaches.**  $S_5$  is a discrete symmetry (not continuous Peccei–Quinn  $U(1)$ ), so it is not broken by gravitational effects, is not subject to quantum anomalies, and is an exact symmetry of the lattice measure at all scales. No axion field is required.

**Lemma 20.1** (Stability). *Let  $\mu$  be any  $S_5$ -invariant measure,  $Q$  in the sign representation. Then  $\int Q d\mu = 0$ .*

**Corollary 20.2.** *If  $S_{\text{eff}} = S_0 + \sum c_n O_n$  where every  $O_n$  is an  $S_5$ -invariant counterterm of arbitrary dimension, then  $e^{-S_{\text{eff}}}|d\theta||d\varphi|$  is  $S_5$ -invariant  $\Rightarrow \langle Q \rangle = 0$  for any  $\{c_n\}$ . The protection is absolute. The only escape is an operator breaking  $S_5$ , but  $S_5 = \text{Aut}(K_5)$  is exact.*

**Falsifiable prediction.** Neutron EDM from  $\theta_{\text{QCD}} = 0$ . Observable EDM is from the CKM phase only ( $\sim 10^{-31} e \cdot \text{cm}$ ). Detection of a  $\bar{\theta}$ -induced EDM at levels incompatible with CKM would falsify the mechanism.

*Gap closure.*  $S_5$  acts on all sectors. On gauge:  $S = \beta \sum (1 - \cos \Phi)$  depends on  $\cos$  (even), hence invariant. On Higgs: adjoint action depends on traces, hence invariant. On fermions:  $D \rightarrow PDP^{-1}$ , hence  $\det(D)$  invariant. On the measure:  $|\det(\text{permutation})| = 1$ , hence invariant. There is no discrete anomaly:  $S_5$  acts on colour indices by conjugation (vector-like, not chiral), and vector-like discrete symmetries have no 't Hooft anomalies. In the continuum:  $\Phi_f \sim a^2 F_{\mu\nu}$ , so  $Q \sim \varepsilon_{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma} \cdot a^4 = a^4 \text{Tr}(F\tilde{F})$ —quadratic in  $F$ , Levi-Civita contracted, pseudoscalar, covering all 4 directions.

## 20.2 Proton stability

Every known channel of proton decay is blocked. The gauge channel requires  $X, Y$  bosons in  $(3, 2, -5/6)$ —but  $K_5$  induces  $SU(3)$ , not  $SU(5)$ , and these bosons do not exist. The scalar channel requires a coloured Higgs—but the  $K_5$  Higgs is an adjoint EW singlet. The dimension-5 channel requires a colour triplet—none is available. The first allowed operators appear at dimension 6, giving  $\tau_p > 10^{36}$  yr for  $\Lambda = 10^{16}$  GeV.

Observation of  $p \rightarrow \bar{\nu} K^+$  at any lifetime would directly falsify the theory.

## 20.3 No dark-matter particles

$K_5$  produces exactly the Standard Model:  $(\bar{5} \oplus 10) \times 3$ . No additional representations, no hidden sector, no BSM fields. Discovery of a dark-matter particle in direct detection would require extending the minimal framework.

# 21 Non-perturbative sector dynamics

## 21.1 Gauge-fixed propagator

In star gauge ( $\theta_{0i} = 0$ ), 4 edges are fixed and 6 physical edges remain. The action  $S \approx (\beta/2)\theta^\top M\theta$  with  $M$  having eigenvalues  $\{1 (\times 3), 5 (\times 3)\}$ . The two eigenvalues correspond to two irreps of  $A_5$ : 3 (chirality  $L$ , eigenvalue 1) and  $3'$  (chirality  $R$ , eigenvalue 5). Consequence:  $\langle \Phi^2 \rangle = 3/(5\beta)$ —identical on all 10 faces by  $A_5$  symmetry. The face propagator  $P = V^\top M^{-1}V$  is a rank-6 projector: it projects onto the 6-dimensional physical space from the 10-dimensional face space.

## 21.2 One coupling, two sectors

One lattice coupling:  $\alpha = 1/60$ ,  $\beta = 4.775$ —the coupling on the  $K_5$  cell, not  $SU(5)$ -type unification. The EW and colour sectors acquire different effective couplings through breaking and running. By Schur's lemma,  $M|_3 = M|_{3'}$ . But by T7 (§17.2): spinor 2 sees only sector 3 via  $P_3$ , while  $2'$  sees only  $3'$ . With  $N_L \neq N_R$ , emergent  $g_{\text{eff},3}^2 \neq g_{\text{eff},3'}^2$ .

### 21.3 Why the effect is non-perturbative

$\det(D) \in \mathbb{R}$  ( $D^\top = \bar{D} \Rightarrow$  determinant is real,  $\Delta = 8 \times 10^{-16}$  on 5000 configurations).

Hessian T7:  $F_L$  eigenvalues =  $\{-0.723(\times 3), 0(\times 3)\}$ . Non-zero in sector 3, zero in 3'.  $F_L < 0$ : one-loop predicts softening—the wrong sign. MC shows stiffening. The mechanism is fundamentally non-perturbative.

Logarithmic barrier: at  $|\theta| = 0 \rightarrow -0.990$ ;  $|\theta| = 0.5 \rightarrow -0.745$ ;  $|\theta| = 1.0 \rightarrow +0.490$  ( $\det \rightarrow 0$ ). The “dome” confines  $\theta_3$ .  $V_{\text{ferm}} \equiv 0$  along sector 3'. T7 is exact.

### 21.4 MC: sector splitting

Config	$\Phi^2$ ratio	Plaq ratio	Var ratio	$\sigma$
Pure gauge	$1.00 \pm 0.01$	$1.00 \pm 0.01$	1.00	1.8
$1L + 0R$	0.86	0.83	0.72	22
$3L + 0R$	0.69	—	0.44	62
$3L + 1R$ (SM)	0.80	0.81	0.65	30
$0L + 1R$ (mirror)	1.17	1.18	1.45	18

Pure gauge null  $\checkmark$ , monotonicity  $\checkmark$ ,  $L \leftrightarrow R$  mirror  $\checkmark$ , decoupling  $\checkmark$ , stability  $L=1.6 \checkmark$ .

The physical picture: fermions stiffen their own sector ( $\Phi^2$  decreases) and soften the opposite sector ( $\Phi^2$  increases). The SM configuration ( $3L+1R$ ) has a 35% variance asymmetry between sectors—the same  $K_5$  cell, the same  $\beta$ , but one sector is tighter than the other. This is the non-perturbative origin of the effective coupling split between SU(2) and U(1).

### 21.5 Universal equation of state

$\ln(\text{Var}(O_3)/\text{Var}(O_{3'}))_{\text{corr}} = -(D/\beta) \cdot \Delta N$ , with  $D = 1.47 \pm 0.11$  (7%).  $c_3 \cdot \beta = \text{const}$  across 6 values of  $\beta$ . The non-perturbative constant  $D \cdot \beta = 1.47$ .

### 21.6 IR screening

On an  $8 \times 8$  lattice of  $K_5$  cells with inter-cell coupling  $J = 4.0$ , 10 shared faces per bond,  $N_m = 1500$  measurements, 300 bootstrap resamples: baseline-corrected sector ratios: pure gauge = 1.16,  $3L+0R = 0.55$  (53% suppression), SM = 0.70 (30% suppression).

The multi-cell lattice confirms the single-cell pattern: sector splitting persists and grows with the number of cells. The IR sector ratio approaches a fixed point from above, consistent with the universal equation of state.

### 21.7 Infrared Lorentz invariance

A discrete causal window appears, at first sight, to threaten Lorentz invariance. The  $K_5$  construction avoids this in a specific way. Lorentz symmetry is not imposed as a microscopic postulate. It is the infrared symmetry of the surviving gapless transport sector of the ordered causal network.

The mechanism has three ingredients.

*First*, the causal arrow is primitive, but the Wilson action on the local unoriented  $K_5$  window does not hard-code a preferred spatial axis. After a past face is selected, the remaining non-past faces form the tetrahedral  $A_4$ -symmetric local structure: one continuation channel and three spatial channels. The ordered background selects a continuation branch, while the local fluctuation operator is still governed by the symmetric gluing data of the window.

*Second*, the transfer operator across a shared  $K_4$  seam separates the long-distance transport sector from heavy subleading sectors. The dominant phase-non-equivalence response mode is transmitted without a mass gap ( $\sigma = 1$ ), whereas the subleading modes acquire finite gaps of

order the microscopic causal scale ( $\sigma = 1/4$ ). In transfer language, if  $\lambda_a(k) = e^{-\varepsilon_a(k)}$ , then only the branch with  $\varepsilon_a(0) = 0$  survives at macroscopic distance. Modes with  $\varepsilon_a(0) > 0$  are exponentially suppressed and do not define the observed low-energy light cone.

*Third*, the  $K_5$  window is geometrically frustrated (dihedral angle  $\arccos(1/4) \approx 75.5^\circ$ , non-integer packing) and does not generate a regular crystal of preferred axes. The mismatch of simplex angles forces successive gluing frames to sample orientations rather than accumulate a fixed lattice anisotropy. This suppresses the usual preferred-axis artefacts of regular lattices. In the infrared, the surviving massless phase-response sector therefore sees an effectively rotational continuum with universal causal speed  $c = 1$  in lattice units.

Thus Lorentz invariance in  $K_5$  is not exact microscopic lattice symmetry. It is the emergent kinematics of the unique gapless transport sector after gapped anisotropic modes and microscopic preferred-frame data have decoupled.

**NLO transfer expansion.** The explicit  $K_4$  transfer calculation gives

$$\sigma(T_{K_4}) = \{1, 1/4, 1/4\}.$$

The heavy seam modes therefore have transfer gap

$$\mu_{\text{sub}} = -\log(1/4) = \log 4.$$

They do not generate a second long-distance light cone; they only contribute local higher-derivative corrections after being integrated out.

In the mature synchronized phase the three spatial channels have been abelianized, so the bulk seam symbol is the nearest-neighbour three-channel symbol

$$\lambda(q) = 2 \sum_{i=1}^3 (1 - \cos q_i).$$

For small  $q$ ,

$$\lambda(q) = q^2 - \frac{1}{12} \sum_i q_i^4 + O(q^6).$$

The cosine symbol is even, so there is no linear  $O(q)$  or cubic odd-power correction. In particular, the minimal mature vacuum contains no linear  $E/E_{\text{cell}}$  Lorentz-violation term.

If the microscopic gluing frames were fixed, the quartic term would retain the preferred-axis invariant  $\sum_i q_i^4$ . The  $K_5$  frustration mechanism averages the orientations of successive seams. For an isotropic orientation average,

$$\left\langle \sum_i n_i^4 \right\rangle = \frac{3}{5}, \quad n_i = q_i/|q|,$$

and hence

$$\lambda_{\text{avg}}(q) = q^2 - \frac{q^4}{20} + O(q^6),$$

so that the phase speed of the surviving branch is

$$\frac{\omega(q)}{|q|} = 1 - \frac{q^2}{40} + O(q^4)$$

in lattice units.

Thus the leading functional form of residual LIV in the mature vacuum is quadratic in the ratio of energy to the microscopic scale, not linear. The universal finite-algebra result is the absence of an  $O(E/E_{\text{cell}})$  term and the existence of a gapped subleading sector with gap  $\log 4$ . The remaining precision target is the small anisotropic residue controlled by deviations from perfect frustrated orientation averaging; that coefficient is the quantity to confront with high-energy photon, neutrino and cosmic-ray observations [35].

## 22 Gravity

### 22.1 Gravity as network elasticity

A massive particle is a graph defect. Around the defect the graph is “compressed”  $\rightarrow$  geodesic motion = gravity. The chain: Ollivier–Ricci curvature [18] on the graph  $\rightarrow$  Regge action [5] on the triangulation  $\rightarrow$  Einstein–Hilbert action (unique by Lovelock’s theorem [6] in  $d = 4$ )  $\rightarrow$  Einstein equations  $G_{\mu\nu} = 8\pi G T_{\mu\nu}$ .

**Theorem 22.1** ( $K_5$  curvature). *A regular lattice of identical equilateral 4-simplices cannot be exactly flat. The dihedral angle of the regular 4-simplex is  $\alpha = \arccos(1/4) \approx 75.52^\circ$ . For zero deficit around a hinge one needs  $2\pi/\alpha \approx 4.77$  simplices—not an integer. Curvature is not an added ingredient but a geometric consequence of regular  $K_5$  gluing.*

Curvature-sign lemma: at  $n_h = 4$ , deficit  $\varepsilon = +1.011$  rad (positive); at  $n_h = 5$ ,  $\varepsilon = -0.307$  rad (negative). Flatness is the effective average over an irregular ensemble.

Regge action:  $S_{\text{Regge}} = (1/16\pi G) \sum_h A_h \varepsilon_h$ . For the regular 4-simplex with edge  $a$ :  $A_h = (\sqrt{3}/4)a^2$ . Effective Newton coupling from matching Regge action with Wilson gluing:  $G = \sqrt{3}/(32\pi\beta)$ .

*Planck length from  $K_5$ .* From  $G \propto a^2/\beta$ :  $\ell_P = a\sqrt{\sqrt{3}/(32\pi\beta)}$ . Numerically:  $\ell_P/a \approx 0.059$  ( $\beta = 5$ ), 0.042 ( $\beta = 10$ ), 0.017 ( $\beta = 60$ ). The  $K_5$  cell is substantially larger than the Planck length.  $\ell_P$  is an emergent scale from gluing and  $\beta$ , not a cell size. At large  $\beta$  the continuum limit is automatic.

*Why gravity is weak.*  $F_{\text{grav}}/F_{\text{em}} = \alpha^{-1} \cdot (m_e/M_{\text{Pl}})^2 \approx 3 \times 10^{-43}$  (experiment:  $2.4 \times 10^{-43}$ ). The electron is  $\exp(-51.5) = 22$  orders below Planck; the square gives 43.

*Hierarchy of masses.* One-loop SU(3) running from  $\alpha^{-1} = 60$ :  $(M_p/M_{\text{Pl}})_{\text{baseline}} \approx 7.6 \times 10^{-17}$ . Physical:  $7.7 \times 10^{-20}$ . Seventeen of twenty orders are automatic; the remaining three are within two-loop and threshold matching.

*Defect curvature response.* A Type B defect creates a curvature cloud  $\Delta\bar{\varepsilon}(r)$  with two regimes: gauge-mediated ( $r < 1.6$ , exponential decay  $\exp(-\sqrt{5}r)/r$ ) and gravity-mediated ( $r > 1.6$ , power-law  $GM/r^2$ ). Pair potential of two defects:  $V(r) = -0.032/r^{1.15}$ , consistent with Newtonian  $-GM^2/r$  ( $p = 1.15 \pm 0.15$ ). Two independent methods (Hessian determinant ratio and curvature integral) agree to 10%. Universality: at  $r \gg 1$  only mass matters—point and distributed sources produce the same curvature (0.6% difference at  $r = 8$ ).

*Emergent gravity: gauge vs collective modes.* The 6 physical edge DOF per  $K_5$  cell have mass gap  $\sim 5.0$  (all nonzero eigenvalues of the Hessian  $B^\top B$  equal 5.0 exactly, independent of lattice size  $L$ ). These are the gauge bosons. Graph Laplacian cell-adjacency network: spectral gap  $\lambda_1 \sim N^{-1.00} \rightarrow$  massless collective modes in the IR = geometric deformations. Clear scale separation: gauge (gapped  $\sim 5$ ) vs gravity (gapless).

Discrete Gauss on the lattice (partition of unity):  $\alpha_h = 1/|D_h|$  is the unique local weight at which  $\sum_\sigma [\sum_{h \subset \sigma} \alpha_h f(h)] = \sum_h f(h)$ . All 10/10 hinges: PoU = 1.000 (exact).

### 22.2 $V_0 = 1/(10\pi)$ : Newton coupling from $K_5$ Green function

**Theorem 22.2.** *The long-range potential between two closure defects on a  $K_5$  lattice:  $V(r) = -V_0/r^p$ , with*

$$V_0 = \beta \times g^2 \times C_{\text{grav}} = \frac{1}{10\pi}, \quad (39)$$

where  $\beta = |A_5|/(4\pi) = 15/\pi$ ,  $g = \text{Tr}(\delta H)/\text{Tr}(H_{\text{vac}}) = 1/5$ ,  $C_{\text{grav}} = 1/\dim_{\text{phys}} = 1/6$ .

Derivation. A 3D  $K_5$  lattice  $L \times L \times L$  is constructed with causal gluing along three spatial directions via tets  $\{2, 3, 4\}$  ( $\text{tet}_2 = \{0, 1, 3, 4\}$ ,  $\text{tet}_3 = \{0, 1, 2, 4\}$ ,  $\text{tet}_4 = \{0, 1, 2, 3\}$ ). The gluing action through  $B_{\text{shared}}^\top B_{\text{shared}} = \{0, 0, 0, 4, 4, 4\}$ : 3 physical restoring + 3 gauge zeros per shared

tetrahedron. The source is an  $S_5$ -averaged  $\delta H$  with diagonal =  $3/5$  (isotropic). The Green function  $G = L^+$  (pseudoinverse of the vacuum Hessian) gives the pair potential  $V(d) = -s_0^\top G s_d$  between two closures at separation  $d$ .

The physical form:

$$V_0 = \beta \times g^2 \times C_{\text{grav}} = \frac{15}{\pi} \times \frac{1}{25} \times \frac{1}{6} = \frac{15}{150\pi} = \frac{1}{10\pi} = 0.03183. \quad (40)$$

Measured from 2-defect potential (§22.1):  $V_0 \approx 0.032$ . Agreement 0.5%.

All numbers are  $K_5$ -native:

Number	Value	Origin
$ A_5 $	60	order of alternating group
$4\pi$	—	Gaussian normalisation of gauge coupling
$\beta$	$ A_5 /(4\pi) = 15/\pi$	inverse bare coupling
$\text{Tr}(\delta H)$	6	trace of $S_5$ -invariant closure perturbation
$\text{Tr}(H_{\text{vac}})$	30	trace of vacuum Hessian on physical subspace
$g$	$\text{Tr}(\delta H)/\text{Tr}(H_{\text{vac}}) = 1/5$	gravitational charge of closure
$\text{dim}_{\text{phys}}$	6	physical modes per cell
$C_{\text{grav}}$	$1/\text{dim}_{\text{phys}} = 1/6$	from lattice Green function

$C_{\text{grav}}$  convergence by lattice size:

$L$	$N_{\text{cells}}$	DOF	$V_0(\text{bare})$	$C_{\text{grav}}$
5	125	1250	0.00761	0.190
7	343	3430	0.00686	0.171
9	729	7290	0.00670	$0.167 \approx 1/6$

$C_{\text{grav}}$  converges to  $1/6 = 1/\text{dim}_{\text{phys}}$ —each of the 6 physical modes per cell contributes equally to the gravitational response.

Consequences: the gravitational normalisation fixes the cell energy relative to the Planck scale,

$$E_{\text{cell}} = \frac{M_{\text{Pl}}}{\sqrt{10\pi}}.$$

The Planck scale itself is not an independent calibration in the updated one-scale web: it is reconstructed from the electron anchor by

$$M_{\text{Pl}} = m_e \exp\left(51 + \frac{19}{36}\right)$$

(§26.12). Thus  $V_0 = 1/(10\pi)$  supplies the gravitational quotient  $C_{\text{grav}} = 1/6$  and the cell/Planck conversion, while the absolute Planck scale is fixed by the projection–gravity bridge.

### 22.3 Einstein–Hilbert length and gauge-gravity crossover

**Theorem 22.3** (Algebraic identity).

$$\boxed{16\pi \cdot V_0 = \frac{8}{5}}. \quad (41)$$

*Proof.*  $V_0 = 1/(10\pi)$  from §22.2. Then  $16\pi V_0 = 16\pi/(10\pi) = 16/10 = 8/5$ .  $\square$

Triple decomposition:

$$\frac{8}{5} = 16\pi V_0 = \frac{4|A_5|}{|V(K_5)|^2 \cdot \dim_{\text{phys}}} = \frac{\dim \mathfrak{su}(3)}{|V(K_5)|}. \quad (42)$$

Checks: combinatorial  $4 \cdot 60/(5^2 \cdot 6) = 240/150 = 8/5$ ; structural  $\dim \mathfrak{su}(3)/|V(K_5)| = 8/5$ .

Physical meaning:  $16\pi$  is the normalisation coefficient of the Einstein–Hilbert action  $S_{EH} = (1/16\pi G) \int R \sqrt{-g} d^4x$ . Thus  $16\pi V_0$  is the natural length scale at which the EH coefficient becomes  $O(1)$  in lattice units—the discrete EH length of  $K_5$ .

*Algebraic bridge between §22.1 and §22.2.* Before this result, the theory contained two independent gravitational objects: the Regge-level Newton coupling  $G = \sqrt{3}/(32\pi\beta)$  from cell geometry (§22.1) and the Green-function coupling  $V_0 = 1/(10\pi)$  from the pair potential (§22.2). The identity  $16\pi V_0 = 8/5$  connects them: it shows that the Einstein–Hilbert normalisation of the first equals the gauge-algebra dimension per vertex of the second. The two are not independent descriptions of different physics but two views of the same structural fact.

*Gauge-gravity crossover.*  $r < r_{EH} = 8/5$ : gauge-mediated,  $V \sim \exp(-\sqrt{5}r)/r$  (mass gap  $\sqrt{5}$  from  $B^\top B$ ).  $r > r_{EH}$ : gravity-mediated,  $V \sim V_0/r$  (gapless cell Laplacian). Agrees with the measured crossover at  $r \approx 1.6$  (§22.1).

Saturation interpretation:  $8/5 = \dim \mathfrak{su}(3)/|V(K_5)| = 1.6$  gauge adjoint modes per vertex. Below  $r_{EH}$ , gauge modes suffice for local response. Above  $r_{EH}$ , they are exhausted (8 adjoint modes distributed over  $\gg 5$  vertices), and the response transitions to the collective regime. This explains why gravity is weak at short distances and dominant at long—not from running couplings, but from gauge-mode saturation.

## 22.4 Black hole as future-sealed causal cluster

### 22.4.1 Minimal sealed region (forced)

**Definition 22.4.**  $R$  is *future-sealed* if for every cell  $C \in R$ , all 4 forward  $K_4$ -faces ( $\text{tet}_0, \text{tet}_1, \text{tet}_2, \text{tet}_3$ ) are glued to cells also in  $R$ .

**Theorem 22.5** (Minimal sealed cluster). *The minimal future-sealed cluster is 4 cells in spatial- $K_4$  configuration (complete graph on 4 vertices). Each  $K_5$  cell has spatial valence 3 (through  $\text{tet}_1, \text{tet}_2, \text{tet}_3$ ); the minimal closed 3-regular graph is  $K_4$ ; all 3 spatial neighbours of each cell must be in  $R$ . Temporal closure is automatically ensured by the sliding quotient.*

**Theorem 22.6** (BH-overclosure seed). *In an oriented sliding  $K_5$  window, the  $k = 4$  overclosure motif is the unique minimal local seed of a future-sealed black-hole cluster. Its physical Hessian has spectrum*

$$\text{spec}_{\text{phys}}(H_4) = \{6, 6, 6, 10, 10, 10\},$$

*so it has full physical rank 6/6. No  $k \leq 3$  motif can be future-sealed, while  $k = 5$  is the saturated vacuum rather than a boundary seed.*

*Proof.* After a past face has been fixed by the sliding orientation, a cell has four forward  $K_4$  faces. Future sealing requires all four of these forward faces to be glued to cells still inside the sealed region. A local motif with  $k < 4$  leaves at least one forward face unglued and therefore violates the definition of future-sealed. The case  $k = 5$  fills the past face as well and removes the exterior boundary distinction; it is the saturated vacuum cell, not a minimal horizon seed. Hence the first possible local seed is exactly  $k = 4$ .

The spectral statement is the  $k$ -tet hierarchy applied at  $k = 4$ . Equivalently, by  $k \leftrightarrow 5 - k$  duality on the physical subspace,

$$H_4 = 10I_6 - H_1.$$

Since  $H_1$  has spectrum  $\{4, 4, 4, 0, 0, 0\}$ ,  $H_4$  has spectrum  $\{6, 6, 6, 10, 10, 10\}$  and therefore full rank. Gluing four such local seeds in spatial- $K_4$  configuration gives the minimal global future-sealed cluster of Theorem 22.5. Thus  $k = 4$  is not a particle sector: it is the local algebraic signature of the minimal black-hole sealed-cluster seed.  $\square$

### 22.4.2 Horizon worldtube (3D)

The horizon  $\partial R_*$  is a 3D boundary worldtube  $\mathcal{H}$ . For the minimal BH ( $R_*^{\min}$ : 4 cells,  $K_4$  config), vertex identifications from spatial gluing give 5 distinct horizon vertices.  $\mathcal{H} = \partial K_5 \setminus \{K_4\} \cong D^3$  (3-ball);  $\partial K_5$  is  $S^3$  (boundary of 4-simplex); removing one  $K_4$  face gives the 3-ball.

### 22.4.3 Forced 2D horizon cross-section

After temporal sliding quotient:  $\Sigma_H = \mathcal{H}/\sim_t$  is a tetrahedral 2-sphere:  $N_2 = 4$ ,  $N_1 = 6$ ,  $N_0 = 4$ ,  $\chi = 2$ . The 4 triangular faces of  $\Sigma_H$  are the 4 exposed faces of the missing  $K_4$  in  $\partial K_5$ . Two-dimensionality of the BH horizon is forced topology, not an import from continuum GR.

### 22.4.4 Local patch data

For one boundary  $K_4$ -patch:

Quantity	Value	Origin
Raw relational alphabet	$L^*(K_4) = 11$	$\dim \Omega^{\geq 1}(K_4) = 6 + 4 + 1$
Propagating gauge-reduced modes	$d_{\text{patch}} = 3$	$B^\top B$ spectrum $\{4, 4, 4, 0, 0, 0\}$
Geometric quotient factor	$\gamma_{K_5} = \sqrt{2}/4$	projection of $\text{tet}_i$ under sliding

The factor  $\gamma_{K_5}$  is computed analytically: the projection of a spatial tetrahedron  $\text{tet}_i = \{v_0, v_2, v_3, v_4\}$  along the temporal direction  $v_0 \rightarrow v_4$  yields an isosceles triangle with sides  $\{\sqrt{3}/2, \sqrt{3}/2, 1\}$  and area  $\sqrt{2}/4$ . This is  $S_3$ -invariant (identical for all three spatial tetrahedra  $\text{tet}_{1,2,3}$ ).

### 22.4.5 Shell DOF count

On the horizon 2-sphere  $\Sigma_H$  with triangulation  $(N_0, N_1, N_2)$ , the physical number of U(1) gauge modes:

$$D_{\text{shell}} = N_1 + N_2 - N_0 = 2(N_2 - 1). \quad (43)$$

From Euler  $\chi = N_0 - N_1 + N_2 = 2$  on  $S^2$ —an exact topological identity for any triangulation. Minimal BH ( $N_2 = 4$ ):  $D_{\text{shell}} = 6$ . Area law emerges:  $\sigma = D/N_2 \rightarrow 2$  for large  $N_2$ .

### 22.4.6 Planck BH identity

$$A_{\text{Planck BH}} = 16\pi\ell_P^2 = 16\pi/(10\pi) = 8/5 = r_{EH}. \quad (44)$$

The minimal BH has a horizon precisely at the length that  $K_5$  independently identifies as the gauge-gravity crossover.

### 22.4.7 Entropy—explicit $K_5$ -native derivation

For BH in  $K_5$ , horizon gauge-field modes form a closed U(1) gauge theory on the tetrahedral 2-sphere with  $K_5$ -forced parameters:  $|A_5| = 60$  (phase discretisation),  $\beta = |A_5|/(4\pi) = 15/\pi$  (Wilson coupling).

The gauge-mode entropy in the canonical ensemble:  $S_{\text{BH}}^{\text{gauge}} = \log Z_{\text{shell}}(\beta) + \beta\langle S_W \rangle$ .

**Partition function via Fourier (exact).** For the tetrahedral 2-sphere:  $Z_{\text{tetra}} = (1/|A_5|) \sum_{k=0}^{|A_5|-1} [\hat{w}(k)]^4$ , where  $\hat{w}(k) = \sum_F \exp(-\beta(1 - \cos(2\pi F/|A_5|))) \exp(-2\pi i k F/|A_5|)$  is the DFT Wilson weight.

For arbitrary triangulation of  $S^2$  with  $N_2$  faces:

$$Z_{\text{shell}}(N_2) = \frac{1}{|A_5|} \sum_k [\hat{w}(k)]^{N_2}. \quad (45)$$

An exact  $K_5$ -native formula from Wilson action + closed 2-sphere gauge theory.

**Minimal BH ( $N_2 = 4$ ):**

$$S_{\text{BH}}^{\text{gauge}}(\text{Planck}) = 8.122 \text{ nat} = 11.72 \text{ bits}. \quad (46)$$

From exact discrete Fourier sum.

**Closed asymptotic form for  $s_*$  (large  $N_2$ ).** The  $k = 0$  Fourier mode dominates. Bessel expansion:

$$s_*^{\text{asympt}} = \ln |A_5| + \ln I_0(\beta) - \beta \frac{I_1(\beta)}{I_0(\beta)}, \quad (47)$$

where  $I_n$  are modified Bessel functions. At  $K_5$  parameters:  $s_*^{\text{asympt}} = 4.0943 + 3.1040 - 4.2393 = 2.959$  nat per patch.

Components ( $K_5$ -native interpretation):  $\ln |A_5| = 4.094$ : classical phase space (60 discrete states per face);  $\ln I_0(\beta) = 3.104$ : Bessel correction from Wilson coupling dressing;  $\beta I_1/I_0 = 4.239$ : mean Wilson action per face at thermal equilibrium.

The Bessel form uses the continuum limit  $\hat{w}(0) \approx |A_5| \cdot e^{-\beta} \cdot I_0(\beta)$ . The exact discrete Poisson-resummed identity includes sub-leading terms:  $\hat{w}(0) = |A_5| \cdot e^{-\beta} \cdot \sum_{m \in \mathbb{Z}} I_{60m}(\beta)$ . At  $K_5$  parameters, the sub-leading correction  $2I_{60}(\beta)/I_0(\beta) \approx 5.6 \times 10^{-61}$ —numerically negligible. The Bessel formula and the exact discrete sum agree to machine precision ( $\sim 10^{-16}$ ). Formally, the Bessel expression is an asymptotic closed form, not a literal exact identity for  $|A_5| = 60$ .

**Area law (exact structural result):**  $S_{\text{BH}}^{\text{gauge}}(N_2) = s_* \cdot N_2 + O(\log N_2)$ . Linear in  $N_2$  from dominance of  $\hat{w}(0)^{N_2}$  over subleading Fourier modes—forced  $K_5$  structure, not imported.

$N_2$	$S_{\text{BH}}^{\text{gauge}}$ (nat)	$S/N_2$	$S/(N_2-1)$
4 (Planck)	8.122	2.031	2.707
8	19.597	2.450	2.800
20	54.822	2.741	2.885
100	291.805	2.918	2.948
$\infty$	$\infty$	2.959	2.959

**Internal audit (three checks passed).** (1) Topological universality:  $Z_{\text{shell}}$  depends only on  $N_2$ , not on the specific triangulation. Verified for  $N_2 \in \{4, 5, 6, \dots, 1000\}$ . (2)  $\beta$  limits:  $\beta \rightarrow 0 \Rightarrow s_* \rightarrow \ln |A_5| = 4.094$  (uniform phase space, correct high- $T$  limit); at  $K_5$  physical  $\beta$ :  $s_* = 2.959$ ;  $\beta \rightarrow \infty$ : well-defined due to discrete  $|A_5|$  cutoff. (3) Consistency with forced geometry: partition function computed on  $\Sigma_H = \mathcal{H}/\sim_t$ , exactly the quotient geometry forced by §22.4.

Gauge-sector BH entropy [10, 31] is closed. The unified gauge+matter interpretation is strongly supported but formally interpretive: in  $K_5$  there is no separate matter field—defect patterns are specific phase configurations already included in the Wilson sum. For extreme large BH with matter-dominated regime, explicit defect counting remains open.

## 22.4.8 Black-hole information

In  $K_5$ , a black hole is a future-sealed causal cluster rather than a singularity inside a pre-existing continuum. The horizon is the boundary of allowed continuation: external observers can access

only boundary-compatible histories. The entropy computed in the  $K_5$  gauge sector is therefore a count of admissible boundary configurations, not evidence for microscopic information destruction.

This does not yet give a full evaporation calculation. The  $K_5$  statement is more limited and more precise: information is not lost by being sent to a mathematical singularity; it is encoded in the boundary history of a sealed causal region. A full evaporation map would require the time-dependent transition kernel of a shrinking sealed cluster and remains a future calculation.

## 23 Cosmology

### 23.1 Cosmological constant

Why is the cosmological constant so small?

On a single  $K_5$  cell, the answer is exact:  $\Lambda_{\text{local}} = 0$ . Every local balance is perfect—self-dual matches anti-self-dual ( $3 = 3$ ), edges match faces ( $10 = 10$ ), gauge matches Bianchi ( $4 = 4$ ), physical degrees of freedom match roots ( $6 = 6$ ). No local mismatch contributes to vacuum energy.

The non-zero cosmological constant arises globally, from the  $\mathbb{Z}_3$  structure of the collective vacuum on a finite causal domain. This is not a Casimir effect (which gives  $\rho \sim 1/L$ ) but an operator-level split.

### 23.2 Operator theorem: $\Omega_\Lambda = 2/3$ from $\mathbb{Z}_3$ structure

Dependencies: §16.1 ( $K_5$  cell algebra,  $\{0^4, 5^6\}$  spectrum), §18.4 ( $\mathbb{Z}_3 \subset A_4$ ), §22.1 (emergent gravity), §22.2 ( $V_0 = 1/(10\pi)$ , three spatial channels via  $\text{tet}_{2,3,4}$ ).

#### 23.2.1 Operator identification

The question is: what operator on the  $K_5$  lattice corresponds to the vacuum energy density? It cannot be a local operator (§23.1:  $\Lambda_{\text{local}} = 0$ ). It must be a collective quantity—a property of the finite-size region, not of any individual cell.

The collective vacuum free-energy operator on a finite 3D  $K_5$  causal region  $R$  of size  $L^3$  cells (periodic or sealed BC):

$$L_{\text{coll}}^{(R)} = H_{\text{Wilson}}|_{\text{physical}}, \quad (48)$$

where  $H_{\text{Wilson}}$  is the Wilson Hessian on edge-phase space (10 DOF/cell) projected onto the physical subspace (removing 4 gauge zeros per cell). The remaining 6 physical modes per cell have the infinite-volume spectrum  $\{5^6\}$ .

This operator enters the vacuum free energy:

$$F_{\text{vac}}(R) = -\frac{1}{2} \log \det L_{\text{coll}}^{(R)}, \quad Z_{\text{vac}}(R) = (\det L_{\text{coll}}^{(R)})^{-1/2}. \quad (49)$$

The vacuum energy is not the zero-point energy of individual oscillators (which would give a quartically divergent  $\Lambda$ ). It is the log-determinant of the collective Hessian—a finite, well-defined quantity on any finite region.

#### 23.2.2 $\mathbb{Z}_3$ projectors

$\mathbb{Z}_3$  generator:  $\sigma = (v_2 v_3 v_4)$ , fixing  $v_0, v_1$ . Under  $\sigma$ , spatial tets permute cyclically:  $\text{tet}_2 \rightarrow \text{tet}_3 \rightarrow \text{tet}_4 \rightarrow \text{tet}_2$ —the unique  $\mathbb{Z}_3 \subset A_4$  compatible with spatial gluing  $\{2, 3, 4\}$ .

Signed edge permutation:  $P_\sigma$  acts on 10 edge phases with orientation signs. Projectors:

$$P_a = \frac{1}{3} \sum_{n=0,1,2} \bar{\omega}^{an} P_\sigma^n, \quad a \in \{1, \omega, \omega^2\}, \quad \omega = e^{2\pi i/3}. \quad (50)$$

Properties (computationally verified):  $P_\sigma^3 = I$ ;  $P_a^2 = P_a$ ,  $P_a P_b = 0$  for  $a \neq b$ ,  $\sum P_a = I$ ;  $[H, P_\sigma] = 0$  (single cell, periodic lattice, sealed lattice). Independent of the choice of  $\mathbb{Z}_3$  generator in  $A_4$ : all 4 conjugate subgroups give the same partition.

On a 3D cubic  $L^3$  lattice,  $P_\sigma$  acts as local  $\sigma$  on each cell  $\times$  cyclic permutation of spatial coordinates  $(i, j, k) \rightarrow (k, i, j)$ —an exact symmetry of the Wilson Hessian.

### 23.2.3 Operator-level $\mathbb{Z}_3$ split

**Theorem 23.1.** *On the physical subspace of a single  $K_5$  cell, the Wilson Hessian satisfies:*

$$\dim(P_1) = \dim(P_\omega) = \dim(P_{\omega^2}) = 2, \quad 2 \oplus 2 \oplus 2. \quad (51)$$

*On a finite causal region  $R$ :  $[H_R, P_\sigma] = 0$  exactly for periodic and  $\mathbb{Z}_3$ -symmetric sealed BCs.  $H_R$  block-diagonalises into three  $\mathbb{Z}_3$  sectors.*

Coherent/localised separation: coherent  $\sigma$ -symmetric currents live ONLY in the trivial sector ( $|P_\omega J^{\text{coh}}| = |P_{\omega^2} J^{\text{coh}}| = 0$  exactly). Localised  $\sigma$ -breaking fluctuations live in non-trivial sectors. This differentiates “coherent matter” and “vacuum background” at the operator level.

CP symmetry: spectra  $H|_{P_\omega}$  and  $H|_{P_{\omega^2}}$  are identical exactly at any  $L$  (verified for  $L = 1, 2, 3, 4, 5$ ).

### 23.2.4 Thermodynamic limit

**Theorem 23.2.** *On a 3D periodic  $K_5$  lattice  $L^3$ , the vacuum free energy splits:*

$$F_1 : F_\omega : F_{\omega^2} \rightarrow 1 : 1 : 1 \quad \text{as } L \rightarrow \infty. \quad (52)$$

*$k$ -space fixed-point argument.* On a translationally invariant lattice,  $H$  diagonalises in  $k$ -space (Brillouin zone).  $\mathbb{Z}_3$  action:  $(k_x, k_y, k_z) \rightarrow (k_z, k_x, k_y)$ . Generic orbits (size 3,  $k_x, k_y, k_z$  not all equal): 3 eigenvectors form the regular representation  $\mathbb{Z}_3 = \text{trivial} \oplus \omega \oplus \omega^2$ ; each character receives exactly one mode with the same eigenvalue. Fixed points ( $k_x = k_y = k_z$ ): 1D diagonal in 3D Brillouin zone, measure zero. Bulk free-energy density:  $f_a = \frac{1}{3} f_{\text{bulk}}$  for  $a \in \{1, \omega, \omega^2\}$ . Finite-size correction:  $|F_a/F_{\text{total}} - 1/3| = O(L^{-2})$ .  $\square$

Numerical confirmation: extra trivial modes in  $P_1$  vs  $P_\omega$  follow the pattern  $\text{extra} = L - 1$  exactly (matching non-zero diagonal  $k$ -points):

$L$	extra modes	predicted ( $L-1$ )	dev $\times L^2$
1	0	0	—
2	1	1	0.08
3	2	2	0.08
4	3	3	0.08

Deviation  $\times L^2 \rightarrow \text{const} \approx 0.08$  confirms  $1/L^2$  scaling.

### 23.2.5 Boundary insensitivity

The result does not depend on the choice of boundary conditions. On a  $\mathbb{Z}_3$ -symmetric sealed region (Dirichlet BC on all 3 spatial faces symmetrically):  $[H_{\text{sealed}}, P_\sigma] = 0$  exactly, because the Dirichlet walls at  $i = L-1$ ,  $j = L-1$ ,  $k = L-1$  are  $\mathbb{Z}_3$ -equivalent under the cyclic permutation  $x \rightarrow y \rightarrow z$ .  $F_\omega = F_{\omega^2}$  exactly at all  $L$ .  $|F_a/F_{\text{total}} - 1/3| = O(L^{-2})$ —the same scaling as periodic, and the same constant ( $\approx 0.08$ ). The vacuum fraction  $\Omega_\Lambda^{\text{vac}} = 2/3$  is independent of boundary conditions.

### 23.2.6 Physical consequence: $\Omega_\Lambda = 2/3$

The vacuum fraction:

$$\Omega_\Lambda^{\text{vac}} = \frac{F_\omega + F_{\omega^2}}{F_{\text{total}}} \rightarrow \frac{2}{3} \quad (L \rightarrow \infty). \quad (53)$$

Combined with three spatial channels and Planck normalisation  $M_{\text{Pl,red}}^2 = M_{\text{Pl}}^2/(8\pi)$ :

$$\rho_\Lambda = \Omega_\Lambda \rho_c = \frac{2}{3} \cdot 3H_0^2 M_{\text{Pl,red}}^2 = \frac{H_0^2 M_{\text{Pl}}^2}{4\pi}. \quad (54)$$

Numerically:  $\rho_\Lambda = H_0^2 M_{\text{Pl}}^2/(4\pi) = 2.46 \times 10^{-11} \text{ eV}^4$  (experiment:  $2.53 \times 10^{-11}$ ,  $\Delta = 2.7\%$ ).

The coefficient  $1/(4\pi)$  is derived:  $2/3 \times 3 \times 1/(8\pi)$ .

## 23.3 Dark matter: operator-level derivation

Minimal  $K_5$  contains no hidden particle dark sector (§20.3). If the dark matter is not a particle, what is it?

The  $K_5$ -native dark-matter mechanism is a primordial frontier relic: a frozen excess of non-luminous causal structure seeded during early network growth. Below we construct the leading  $K_5$ -native operator for a primordial frontier-relic dark sector and obtain the  $\Lambda$ -candidate ratio  $\Omega_{\text{DM}}/\Omega_b \simeq 5.48$  from three operator-level ingredients: the local gluing bias, the dark projector, and the frontier freeze-out operator. The number itself is closed by this operator chain; the remaining open work is the astrophysical realisation of the relic in halos.

### 23.3.1 Growth as thermodynamic selection, not new mechanics

Network growth is not an ad-hoc mechanism added to the theory. The primitive continuation  $K_5^{(n)} \rightarrow K_5^{(n+1)}$  is the sliding window itself. The Wilson action provides a free-energy ranking for admissible gluing moves,  $\Delta F(g)$ . The growth dynamics is simply the repeated thermodynamic selection of these  $K_5$  gluing moves.

### 23.3.2 Late-time no-go and primordial restriction

In a mature, synchronised network, spatial adjacency exists as an established worldtube relation, and temporal sliding merely continues it. The static Hessian around a closure defect has  $\delta D \geq 0$  for all perturbation modes. Pair-potential measurements confirm: the baryon–vacuum interaction is repulsive or null at all tested separations. A late-time baryonic source for dark matter is therefore absent.

The dark excess can only be produced during the primordial frontier era, when unglued “fresh” spatial slots still exist around a baryonic closure. Freeze-out is the moment when the last fresh slot fills and the baryon becomes an interior cell.

### 23.3.3 Local gluing bias from $K_4$ -seam partition functions

A baryonic closure sits in a  $K_5$  window; a new neighbouring cell is glued through a shared  $K_4$  seam. For vertices  $V = \{0, 1, 2, 3, 4\}$  and tetrahedra  $t_a = V \setminus \{a\}$ , a  $k=3$  closure occupies three tetrahedra indexed by  $T \subset V$ ,  $|T| = 3$ . Gluing through spatial channel  $\tau$  uses the shared  $K_4 = t_\tau$ . The four triangular faces  $f_b = t_\tau \cap t_b$  ( $b \neq \tau$ ) each see closure incidence:

$$c_b(T, \tau) = \mathbf{1}_{\tau \in T} + \mathbf{1}_{b \in T}. \quad (55)$$

This gives two channel types. *In-channel* ( $\tau \in T$ ): the shared  $K_4$  enters the closure, face weights  $D_{\text{in}} = (2, 2, 1, 1)$ . *Out-channel* ( $\tau \notin T$ ): the shared  $K_4$  does not enter the closure, face weights  $D_{\text{out}} = (1, 1, 1, 0)$ .

The exact compact U(1) partition function on the  $K_4$ -seam:

$$Z_{K_4}(D_1, D_2, D_3, D_4) = e^{-\beta \sum_f D_f} \sum_{m \in \mathbb{Z}} \prod_{f=1}^4 I_m(\beta D_f), \quad (56)$$

with  $\beta = 15/\pi$ . Defining  $Z_0 = Z(1, 1, 1, 1)$ ,  $Z_{\text{in}} = Z(2, 2, 1, 1)$ ,  $Z_{\text{out}} = Z(1, 1, 1, 0)$ :

$$r_{\text{in}} = Z_{\text{in}}/Z_0 \approx 0.566, \quad r_{\text{out}} = Z_{\text{out}}/Z_0 \approx 2.052. \quad (57)$$

In-channel is suppressed; out-channel is enhanced.

Primitive growth selects one of three spatial channels. In vacuum all three are equal ( $p_{\text{out}}^{(0)} = 1/3$ ). Near a baryon: one out-channel, two in-channels. The outward growth probability:  $p_{\text{out}}^{(B)} = Z_{\text{out}}/(Z_{\text{out}} + 2Z_{\text{in}})$ . The relative enhancement:

$$\eta_B = \frac{3Z_{\text{out}}}{Z_{\text{out}} + 2Z_{\text{in}}} = \frac{3r_{\text{out}}}{r_{\text{out}} + 2r_{\text{in}}} \approx 1.93. \quad (58)$$

Equivalently,  $\Delta\Delta F_B = -\beta^{-1} \ln \eta_B \approx -0.138$ . This is no longer a fit; it is derived from the competition of  $K_4$ -seam partition functions.

### 23.3.4 Dark projector $P_{\text{DM}}$

From the  $\mathbb{Z}_3$ -split (§23.2), the nontrivial sectors  $P_\omega + P_{\omega^2}$  give the vacuum fraction  $\Omega_\Lambda = 2/3$ . Dark matter cannot live in those sectors (it would mix with dark energy). It must live in the matter-like trivial sector  $P_1$ .

The visible baryon is the coherent  $k=3$  closure current  $|B\rangle$  with projector  $P_{\text{coh}} = |B\rangle\langle B|/\langle B|B\rangle$ . Since  $P_1|B\rangle = |B\rangle$ , the dark matter projector is the orthogonal complement within  $P_1$ :

$$P_{\text{DM}} = P_1 - P_{\text{coh}}. \quad (59)$$

Properties of  $P_{\text{DM}}$ : *Nonluminous*:  $J_{\text{em}}P_{\text{DM}} = 0$  (photon U(1)-transport reads coherent charged current;  $P_{\text{DM}}$  is orthogonal to it). *Gravitating*:  $T_{\text{grav}}P_{\text{DM}} \neq 0$  (gravity couples to gluing stress, not to EM current). *Cold*:  $P_{\text{DM}}$  has no coherent massless transport mode; after freeze-out it co-moves with the network ( $w \approx 0$ ).

This is the minimal  $K_5$ -native projector onto a nonluminous, cold, gravitating, matter-like sector.

### 23.3.5 Energy normalisation

Within the baryon-normalised frontier process, the visible coherent branch and the hidden relic branch are two orthogonal projections of one  $P_1$ -matter sector. On the  $P_1$  frontier sector, the primitive gravitational weighting acts as identity:  $\mathcal{M}_E|_{P_1} = I$ , hence  $\chi_E = \text{Tr}(\hat{P}_{\text{DM}}\mathcal{M}_E) = 1$ .

This does not mean “a dark cell has the mass of a proton.” It means: in baryon-normalised frontier relic counting, hidden and visible  $P_1$ -units carry the same primitive gravitational weight.

### 23.3.6 Topological frontier count ( $N_{\text{eff}} = 23$ )

The frontier around a baryon grows along three spatial channels. Shell  $n$ : triples  $(a, b, c)$  with  $a + b + c = n$ ; raw states  $\mathcal{F}_n^{\text{raw}} = \binom{n+2}{2}$ . For shells 1–3: 3, 6, 10.

Shell 4 is the first where  $S_3$ -synchronisation closes the boundary. The projector  $Q_4 = (1/6) \sum_{\pi \in S_3} U_\pi$  on the 15 raw states has rank 4 (orbits  $(4, 0, 0)$ ,  $(3, 1, 0)$ ,  $(2, 2, 0)$ ,  $(2, 1, 1)$ ), matching the 4 boundary patches of the tetrahedral sealed surface.

The frontier freeze-out operator:

$$\mathcal{F}_{\text{fr}} = I_{\mathcal{F}_1} \oplus I_{\mathcal{F}_2} \oplus I_{\mathcal{F}_3} \oplus Q_4, \quad \text{Tr } \mathcal{F}_{\text{fr}} = 3 + 6 + 10 + 4 = 23. \quad (60)$$

### 23.3.7 Rate-independent local yield ( $\varepsilon_0$ )

A fresh frontier slot faces two opening processes: vacuum ( $P_0 = e^{-y}$ ) and baryon-biased ( $P_B = e^{-\eta_B y}$ ). The total-variation distance:

$$\varepsilon_0 = \sup_y (e^{-y} - e^{-\eta_B y}) = \eta_B^{-1/(\eta_B-1)} - \eta_B^{-\eta_B/(\eta_B-1)}. \quad (61)$$

At  $\eta_B = 1.934$ :  $\varepsilon_0 = 1.934^{-1.071} - 1.934^{-2.071} = 0.493 - 0.255 = 0.238$ .

### 23.3.8 Full DM operator and ratio

The complete dark-matter operator:

$$\mathcal{D}_B = \mathcal{F}_{\text{fr}} \otimes \varepsilon(\eta_B) \otimes \hat{P}_{\text{DM}}. \quad (62)$$

Taking the trace in baryon-normalised frontier units:

$$\boxed{\frac{\Omega_{\text{DM}}}{\Omega_b} = \text{Tr } \mathcal{D}_B = 23 \varepsilon(\eta_B) = 23 \left[ \eta_B^{-1/(\eta_B-1)} - \eta_B^{-\eta_B/(\eta_B-1)} \right] \approx 5.48.} \quad (63)$$

where

$$\eta_B = \frac{3 Z_{K_4}(1, 1, 1, 0)}{Z_{K_4}(1, 1, 1, 0) + 2 Z_{K_4}(2, 2, 1, 1)}. \quad (64)$$

Observed (Planck 2018 [11]):  $\Omega_{\text{DM}}/\Omega_b \simeq 5.36$ . Agreement: about 2.2%.

Full causal chain:

causality  $\rightarrow$  DAG  $\rightarrow K_5 \rightarrow k=3$  closure  $\rightarrow K_4$ -seam partition functions  
 $\rightarrow$  channel competition  $\rightarrow \eta_B = 1.93 \rightarrow$  total-variation yield  $\varepsilon_0 = 0.238$   
 $\rightarrow$  frontier shells  $\rightarrow N_{\text{eff}} = 23 \rightarrow \Omega_{\text{DM}}/\Omega_b = 5.48$ .

### 23.3.9 Status

The mechanism is classified as a **DERIVED CANDIDATE LAW (A-)**. The ratio  $\Omega_{\text{DM}}/\Omega_b$  now has a clean  $K_5$  operator derivation:  $\eta_B$  from  $K_4$ -seam partition functions,  $P_{\text{DM}}$  from minimal projector lemma,  $\chi_E$  from  $P_1$ -normalisation,  $N_{\text{eff}}$  from frontier combinatorics.

Remaining for full A: observational halo phenomenology (density profiles, lensing response, clustering, and survival through structure formation). No further numerical tuning of the ratio is introduced at the minimal level.

Falsifier: discovery of a DM particle in direct detection [36] falsifies minimal  $K_5$ .

## 23.4 Expansion of the Universe

### 23.4.1 Causal bias: $1 \rightarrow 4$

The oriented  $K_5$  has 1 past face and 4 future faces. Each future face shares 3 edges with the past face and 3 with each other future face—they are tightly correlated, not independent branches. The  $1 \rightarrow 4$  causal bias is an internal geometric property of the oriented 4-simplex, not an external field.

**Theorem 23.3** (Primitive growth). *The unique growth mode preserving the 3D spatial frame ( $d_1 = 3$ ) and maximising expansion ( $\Delta S_1 = +3$ ) is  $k=1$  attachment through one past face. At  $k=2$ : 1 DOF remains. At  $k=3$ : 0 DOF. Only  $k=1$  gives the full  $1+4$  causal structure  $= S_5 \rightarrow S_4$ .*

### 23.4.2 Free-energy selection rule

For face-based gluing action and canonical channel entropy  $\Omega_k = \binom{5}{k}$ :

$k$	$E_k$	$\Omega_k$	$F_k$	Preference
1	5.78	5	4.17	dominant
2	9.86	10	7.56	$P_2/P_1 \approx 3\%$
3	12.75	10	10.45	$P_3/P_1 \approx 0.2\%$

$F_1 < F_2 < F_3$ : temporal continuation ( $k=1$ ) is thermodynamically preferred at all coupling values. The crossovers  $\Delta F_{2-1} > 0$  and  $\Delta F_{3-1} > 0$  occur for  $\beta > 0.85$  and  $\beta > 0.50$  respectively.

### 23.4.3 Quantitative status

The  $K_5$  framework structurally differentiates the internal  $\Omega_\Lambda^{\text{vac}} = 2/3$  (operator theorem) from observational cosmological coordinates. The first background map is the scale dictionary from the synchronised frontier volume to redshift.

## 23.5 Minimal background observational map

This subsection closes the homogeneous background map. The following CMB operator subsection supplies the formal thermal, opacity, primordial-kernel and line-of-sight map; a precision numerical  $C_\ell$  spectrum still requires a transfer solver and early-frontier amplitudes.

In the mature abelianised spatial phase,

$$F_n = \binom{n+2}{2}, \quad V_n = \binom{n+3}{3}, \quad (65)$$

where  $F_n$  is the shell size and  $V_n$  is the number of synchronised spatial windows inside shell  $n$ . Define the effective scale factor by

$$a_n = a_0 \left( \frac{V_n}{V_{n_0}} \right)^{1/3}. \quad (66)$$

The observational redshift of a photon emitted at frontier depth  $n_e$  and observed at  $n_0$  is

$$1 + z(n_e, n_0) = \frac{a_{n_0}}{a_{n_e}} = \left[ \frac{V_{n_0}}{V_{n_e}} \right]^{1/3}. \quad (67)$$

The corresponding discrete kinematic Hubble rate is

$$H_{\text{kin}}(n + \frac{1}{2}) \tau_{\text{tick}} = \log \frac{a_{n+1}}{a_n} = \frac{1}{3} \log \frac{V_{n+1}}{V_n} = \frac{1}{3} \log \frac{n+4}{n+1}. \quad (68)$$

For large  $n$ ,  $H_{\text{kin}} \tau_{\text{tick}} \sim 1/n$  and  $a_n \sim n$ . This is the kinematic scale/redshift map of the mature synchronised frontier, not yet a dynamical Friedmann equation.

The minimal dynamical background projection uses the operator fractions derived above:

$$\Omega_m = \frac{1}{3}, \quad \Omega_\Lambda^{\text{vac}} = \frac{2}{3}. \quad (69)$$

In the homogeneous isotropic infrared limit of the Regge/EH sector, the corresponding flat background expansion is

$$E_{K_5}^2(z) \equiv \frac{H^2(z)}{H_0^2} = \Omega_m(1+z)^3 + \Omega_\Lambda = \frac{1}{3}(1+z)^3 + \frac{2}{3} \quad (70)$$

(up to the standard radiation term at early times). It gives the parameter-free background consequences

$$q_0 = \frac{1}{2}\Omega_m - \Omega_\Lambda = -\frac{1}{2}, \quad z_{\text{acc}} = \left(\frac{2\Omega_\Lambda}{\Omega_m}\right)^{1/3} - 1 = 4^{1/3} - 1 \simeq 0.587, \quad (71)$$

$$z_{\Lambda m} = \left(\frac{\Omega_\Lambda}{\Omega_m}\right)^{1/3} - 1 = 2^{1/3} - 1 \simeq 0.260, \quad (72)$$

$$H_0 t_0 = \frac{2}{3\sqrt{\Omega_\Lambda}} \operatorname{asinh} \sqrt{\frac{\Omega_\Lambda}{\Omega_m}} = \frac{2}{3\sqrt{2/3}} \operatorname{asinh} \sqrt{2} = 0.93588 \dots \quad (73)$$

The frontier-relic ratio  $R = \Omega_{\text{DM}}/\Omega_b \simeq 5.48$  then fixes the matter split:

$$\boxed{\Omega_b = \frac{\Omega_m}{1+R} \simeq 0.0514, \quad \Omega_{\text{DM}} = \frac{R\Omega_m}{1+R} \simeq 0.282.} \quad (74)$$

This gives the leading homogeneous map

$$\boxed{(n, V_n, \Omega_\Lambda, \Omega_m, R) \mapsto (z, H(z), \Omega_b, \Omega_{\text{DM}}).} \quad (75)$$

Status: this is a **DERIVED BACKGROUND MAP**. It closes the zeroth-order  $H(z)$  relation implied by the  $K_5$  density fractions and the emergent EH/Regge infrared limit. The next subsection constructs the formal CMB operator map (thermal scaling, opacity, visibility, primordial kernel and angular spectra). A precision numerical  $C_\ell$  prediction still requires non-equilibrium recombination, the early-frontier fluctuation normalisation, and a Boltzmann/line-of-sight solve. If future data require  $w(z) \neq -1$ , the  $K_5$  task is to refine the collective-vacuum sector into a running  $\Omega_\Lambda(z)$  rather than to add a new dark-energy particle.

### 23.6 Pair production, dilution, and causal transparency

The coupling  $\beta = 15/\pi$  is fixed. The suppression of pair creation in the mature universe does not come from a time-dependent coupling; it comes from the state of the local causal background. In the mature ordered phase ( $\Phi_f \simeq 0$ ), creating an electron–positron pair requires overcoming a barrier of order  $2S_e$ —exponentially suppressed. In the early frontier, the gluing background is frustrated and carries nonzero local Wilson cost, so the effective pair-creation barrier is much lower. The same fixed Wilson action therefore produces abundant pairs in the early network and essentially none in the mature network.

After baryonic closure freeze-out, the number of surviving charged defects is approximately fixed. The net electron count equals the net proton count: a  $k=3$  closure in the  $K_5$  window leaves a  $k=2$  complementary residual ( $3+2=5$ ) whose ground state is the electron. Continued  $k=1$  growth adds mostly ordered vacuum windows—which cost no local energy ( $\Lambda_{\text{local}} = 0$ ). The density of charged motifs per causal volume decreases. When electrons bind to baryonic closures, the external charged holonomy is screened; the network becomes transparent to coherent U(1) phase non-equivalence transport.

This gives the qualitative  $K_5$ -native origin of dilution and transparency. The quantitative photon–matter opacity and visibility operators are defined in §23.8.

### 23.7 Baryogenesis: CP-odd operator from cyclic non-commutativity

The Wilson action is CP-even:  $S_W(\Phi) = S_W(-\Phi)$ . Therefore a mature Wilson ensemble produces  $k=3$  closures of both orientations at equal rates. Pure Wilson growth cannot generate a net baryon asymmetry.

The minimal  $K_5$ -native CP-odd operator is constructed as follows. After causal orientation ( $A_5 \rightarrow A_4$ ), the  $\mathbb{Z}_3$  factor of  $A_4 \simeq V_4 \rtimes \mathbb{Z}_3$  cycles the three spatial channels with generator  $\sigma = (123)$ . The handedness operator

$$J_{\mathbb{Z}_3} = P_\omega - P_{\omega^2} \quad (76)$$

changes sign under CP ( $\omega \leftrightarrow \omega^2$ ) and distinguishes the two orientations of a  $k=3$  closure:  $B_+ \sim (1, 2, 3)$  vs  $B_- \sim (1, 3, 2)$ .

The three spatial  $K_4$ -seam growth projectors  $P_a = H_{K_4(a)}/4$  ( $a = 1, 2, 3$ ) do not commute. Their cyclic and anti-cyclic assemblies differ:

$$\Delta_{\text{cyc}} = (P_1 P_2 P_3 + P_2 P_3 P_1 + P_3 P_1 P_2) - (P_1 P_3 P_2 + P_3 P_2 P_1 + P_2 P_1 P_3) \neq 0. \quad (77)$$

The CP-even trace vanishes ( $\text{Tr} \Delta_{\text{cyc}} = 0$ ), but the CP-odd projection is nonzero:

$$\mathcal{I}_{CP} = \frac{\text{Tr}(J_{\mathbb{Z}_3} \Delta_{\text{cyc}})}{\text{Tr}(\Sigma_{\text{even}})} = \frac{5\sqrt{3}}{9} \approx 0.962. \quad (78)$$

This is an exact algebraic invariant of the  $K_5$  seam-projector algebra, verified computationally.

The baryogenesis asymmetry is then

$$A_{\text{orient}}(n) = \tanh[\lambda_{\text{fr}}(n) \cdot \kappa_0], \quad \kappa_0 = \text{arctanh}\left(\frac{5\sqrt{3}}{9}\right) \approx 1.975, \quad (79)$$

where  $\lambda_{\text{fr}}(n) \in [0, 1]$  measures how strongly the early non-equilibrium frontier activates this operator. In the mature ordered network  $\lambda_{\text{fr}} = 0$  and  $A_{\text{orient}} = 0$ . The net electron count equals the net surviving proton count by  $3 + 2 = 5$  complementarity of the  $K_5$  window.

The remaining open task is the evaluation of  $\lambda_{\text{fr}}(n)$  from the early frontier transfer kernel.

### 23.8 CMB operator map: radiation, decoupling, perturbations, and spectra

The homogeneous background map closes the zeroth-order relation between synchronized frontier volume and observational expansion. The CMB requires four further ingredients: a radiation/thermal map, a photon-matter decoupling operator, a primordial perturbation kernel, and a line-of-sight spectrum map. This subsection writes these ingredients in the native  $K_5$  language. It closes the *formal operator map*; it does not yet claim a fully numerical first-principles acoustic  $C_\ell$  curve, because that still requires a Boltzmann transfer computation and the exact early-frontier fluctuation amplitudes.

**Radiation and thermal map.** Photons are the massless  $U(1)$  phase-non-equivalence response branch. Their thermal ensemble redshifts with the synchronized frontier scale factor:

$$T(z) = T_0(1+z), \quad \rho_\gamma(z) = \rho_{\gamma 0}(1+z)^4. \quad (80)$$

The radiation density parameter is a thermal anchor,

$$\Omega_{r0} = \Omega_{\gamma 0} (1 + 0.2271 N_{\text{eff}}), \quad (81)$$

where  $T_0$ ,  $H_0$ , and  $N_{\text{eff}}$  are observational calibration inputs unless separately derived from the early frontier sector. Including radiation while preserving the  $K_5$  matter-vacuum ratio gives

$$\Omega_{m0} = \frac{1 - \Omega_{r0}}{3}, \quad \Omega_{\Lambda 0} = \frac{2(1 - \Omega_{r0})}{3}, \quad (82)$$

so the background map becomes

$$E_{K_5}^2(z) = \Omega_{r0}(1+z)^4 + \frac{1 - \Omega_{r0}}{3}(1+z)^3 + \frac{2(1 - \Omega_{r0})}{3}. \quad (83)$$

For late times,  $\Omega_{r0} \ll 1$  and this reduces to the previous  $\frac{1}{3}(1+z)^3 + \frac{2}{3}$  map.

**Photon–matter decoupling operator.** Charged Type-B defects couple to the U(1) response branch. Neutral baryonic closures screen their external charged holonomy. Thus the optical depth is controlled by the unbound Type-B projector, equivalently by the free-electron fraction  $X_e(z)$ :

$$n_e(z) = X_e(z) n_b(z), \quad n_b(z) = \frac{\Omega_{b0}\rho_{c0}}{m_p}(1+z)^3. \quad (84)$$

The Thomson scattering scale is fixed by the same edge-phase coupling and Type-B electron pole,

$$\boxed{\sigma_T = \frac{8\pi}{3} \frac{\alpha^2}{m_e^2}} \quad (\hbar = c = 1). \quad (85)$$

The  $K_5$  decoupling operator is therefore

$$\boxed{\frac{d\tau}{dz} = \frac{\sigma_T n_e(z)}{(1+z)H(z)}, \quad g(z) = e^{-\tau(z)} \left| \frac{d\tau}{dz} \right|}, \quad (86)$$

where  $g(z)$  is the visibility function. The leading equilibrium estimate for  $X_e$  is the Saha relation

$$\boxed{\frac{X_e^2}{1-X_e} = \frac{1}{n_b(z)} \left( \frac{m_e T(z)}{2\pi} \right)^{3/2} \exp\left[-\frac{B_H}{T(z)}\right]}, \quad B_H = \frac{1}{2}\alpha^2 m_e. \quad (87)$$

In  $K_5$  terms, recombination is the binding of the Type-B residual to the baryonic  $k=3$  closure, which projects the external charged holonomy to the neutral sector. Using the  $K_5$  electromagnetic pole values gives

$$B_H = \frac{1}{2}\alpha^2 m_e = 13.606 \text{ eV}, \quad (88)$$

so the leading Saha visibility peak is at

$$z_*^{\text{Saha}} \simeq 1.27 \times 10^3 \quad (89)$$

when the present thermal boundary is anchored to the observed CMB temperature. A precision calculation of  $X_e(z)$  requires the nonequilibrium recombination operator, helium composition and radiative-transfer corrections; these shift the physical last-scattering peak toward the standard lower redshift. The causal mechanism and opacity operator, however, are fixed.

**Primordial perturbation kernel.** Let  $\mathcal{R}$  denote the scalar curvature perturbation of the synchronized frontier geometry. The leading fixed-point kernel has the scale-invariant form

$$\boxed{\langle \mathcal{R}(\mathbf{k}) \mathcal{R}(\mathbf{k}') \rangle = (2\pi)^3 \delta^3(\mathbf{k} + \mathbf{k}') \frac{2\pi^2}{k^3} \Delta_{\mathcal{R}}^2(k), \quad \Delta_{\mathcal{R}}^2(k) = A_s \left( \frac{k}{k_*} \right)^{n_s-1}}. \quad (90)$$

The mature  $K_5$  fixed point gives the scale-free seed  $n_s = 1$  at leading order. The first finite-ledger correction is naturally controlled by the full  $K_5$  ledger size

$$L^* = \binom{5}{2} + \binom{5}{3} + \binom{5}{4} + \binom{5}{5} = 26. \quad (91)$$

This gives the minimal ledger-anomalous candidate

$$\boxed{n_s^{K_5} = 1 - \frac{1}{L^*} = \frac{25}{26} = 0.961538\dots} \quad (92)$$

The amplitude and tensor response are assigned to the next-to-leading frontier fluctuation kernel:

$$A_s = A_{\text{fr}}, \quad r = r_{\text{fr}}. \quad (93)$$

The tilt value above is a **candidate frontier-kernel benchmark**, not a theorem-level CMB fit. A full calculation must derive  $A_{\text{fr}}$  and  $r_{\text{fr}}$  from the non-equilibrium growth ensemble.

**Line-of-sight CMB spectrum map.** Given the background map, decoupling operator, and primordial kernel, the CMB spectra are determined by the line-of-sight response of the same photon–matter system:

$$C_\ell^{XY} = 4\pi \int_0^\infty \frac{dk}{k} \Delta_{\mathcal{R}}^2(k) \Delta_\ell^X(k) \Delta_\ell^Y(k), \quad X, Y \in \{T, E\}. \quad (94)$$

The transfer functions  $\Delta_\ell^X$  are to be computed by the photon–matter Boltzmann transfer system instantiated with the  $K_5$  opacity, background, and primordial-kernel operators above, with sound speed

$$c_s^2(z) = \frac{1}{3[1 + R_b(z)]}, \quad R_b(z) = \frac{3\rho_b(z)}{4\rho_\gamma(z)}. \quad (95)$$

The leading acoustic scale is

$$r_s(z_*) = \int_{z_*}^\infty \frac{c_s(z)}{H(z)} dz, \quad D_M(z_*) = \int_0^{z_*} \frac{dz}{H(z)}, \quad \ell_A = \pi \frac{D_M(z_*)}{r_s(z_*)}. \quad (96)$$

Here  $z_*$  is the peak of the visibility function  $g(z)$ . With the leading Saha peak one obtains  $\ell_A \simeq 3.3 \times 10^2$ ; with the delayed non-equilibrium last-scattering peak near the standard recombination redshift the same background gives  $\ell_A \simeq 3.0 \times 10^2$ . This gives the formal map

$$(\Omega_{\Lambda 0}, \Omega_{m 0}, \Omega_{b 0}, \Omega_{r 0}, X_e(z), \Delta_{\mathcal{R}}^2) \mapsto C_\ell^{TT}, C_\ell^{TE}, C_\ell^{EE}. \quad (97)$$

**Status.** The  $K_5$  framework now supplies a complete formal CMB operator map: radiation redshift, opacity/visibility, primordial-kernel form, and line-of-sight spectrum functional. This closes the formal observational map, not the full numerical acoustic CMB prediction. What remains is not an additional particle sector, but the numerical solution of the photon–matter transfer equations and the evaluation of  $A_{\text{fr}}$  and  $r_{\text{fr}}$  from the early frontier growth ensemble. The finite-depth tilt  $n_s^{K_5} = 25/26$  is a candidate frontier-kernel benchmark. A full numerical acoustic  $C_\ell$  prediction is therefore a **precision benchmark**, not yet a theorem-level result.

## 24 Boundary transition kernels and the S-matrix analogue

In  $K_5$ , scattering is not primitive. The primitive object is a finite causal region  $\mathcal{R}$  with boundary motifs. A scattering experiment prepares stable motifs on the past boundary  $\partial_- \mathcal{R}$  and reads stable motifs on the future boundary  $\partial_+ \mathcal{R}$ .

### 24.1 The boundary-to-boundary kernel and thermodynamic selection

For a finite causal slab  $\mathcal{R}$  (a chain of  $N$  sliding windows), boundary motifs  $\alpha \in \mathcal{H}_{\partial_-}$  and  $\beta \in \mathcal{H}_{\partial_+}$  define the transition amplitude. This kernel should not be read as a sum over arbitrary causal topologies. In the ordered  $K_5$  phase, the gluing skeleton is thermodynamically selected. The primitive  $k=1$  continuation and the aligned shared- $K_4$  gluing strictly minimise the local free energy (§23.4); alternative gluing patterns are suppressed corrections rather than equally real histories.

The leading  $K_5$  transition kernel is therefore a phase response evaluated strictly on this selected causal skeleton  $\mathcal{G}_*$ :

$$K_{\beta\alpha}^{K_5} = \int_{\Theta(\mathcal{G}_*; \alpha, \beta)} \mathcal{D}\theta e^{-S_W[\theta]} U_{\beta\alpha}[\theta]. \quad (98)$$

Here  $\mathcal{G}_*$  is the thermodynamically selected causal gluing skeleton,  $S_W[\theta] = \beta \sum_f (1 - \cos \Phi_f)$  is the real-valued Wilson cost on this skeleton (suppressing high-curvature configurations), and  $U_{\beta\alpha}[\theta] = \prod_{e \in \Gamma_{\beta\alpha}} e^{i\epsilon_e \theta_e}$  is the oriented U(1) phase transporter between the boundary motifs.

In transfer notation:  $K_{\beta\alpha}(N) = \langle \Psi_\beta | T^N | \Psi_\alpha \rangle$ , where  $T$  is the single-step causal transfer operator built from Wilson weights and gluing constraints (§19.8).

This is not yet the S-matrix. It is the transition kernel on the selected skeleton, including free propagation of the external motifs through the slab.

## 24.2 External states: stable motifs

An external state in  $K_5$  is a stable causal motif—an isolated transfer eigenmode with a definite decay exponent and residue:  $T|\psi_A\rangle = \lambda_A|\psi_A\rangle$ , or equivalently  $G_A(n) \sim Z_A e^{-m_A n}$ .

The mass of a motif is its causal decay exponent. Its residue  $Z_A$  normalises its external projection.

The catalogue of admissible external motifs is fixed by  $K_5$ : photon (massless U(1) phase-non-equivalence response mode), electron (Type B charged defect), neutrino (neutral carrier mode), proton ( $k=3$  closure), pion ( $k=2$  bridge resonance), Higgs (radial ordering mode), graviton-like response (collective massless network mode).

## 24.3 Born probabilities from oriented phase gluing

The theory does not arbitrarily introduce a complex amplitude by mathematically transforming  $e^{-S_W}$  into  $e^{iS_W}$ . The thermodynamic and quantum elements are distinct: the Wilson weight  $e^{-S_W}$  provides the thermodynamic suppression of phase configurations, while the edge transporter  $U_{\beta\alpha}$  supplies the complex quantum phase.

The sum is taken over internal phase configurations on the fixed skeleton, not over arbitrary worlds. Interference arises naturally because multiple internal phase configurations on the same causal skeleton can induce the same boundary transition with different accumulated phases. The physical Born probability is the modulus squared of this oriented gluing:

$$P(\beta|\alpha) = \frac{|K_{\beta\alpha}|^2}{\sum_{\beta'} |K_{\beta'\alpha}|^2}, \quad (99)$$

where complex conjugation corresponds strictly to topological orientation reversal ( $g_{e^{-1}} = g_e^{-1} = e^{-i\theta_e}$ ).

This is not the Born rule imported from quantum mechanics. It is the unique probability measure compatible with gauge invariance, positivity, and additivity for decoherent alternatives (§25.1)—here derived from the factored structure of the kernel itself.

## 24.4 Amputation without LSZ import

In the standard framework, the LSZ reduction [14] extracts on-shell amplitudes from correlation functions. In  $K_5$ , the same result is obtained without importing LSZ:

1. Compute the boundary kernel  $K_{\beta\alpha}$ .
2. Project boundary data onto stable eigenmotifs  $P_\alpha, P_\beta$ .
3. Divide by the residues of free propagation:

$$\mathcal{A}_{\beta\alpha}^{K_5} = Z_\beta^{-1/2} Z_\alpha^{-1/2} P_\beta K_{\beta\alpha}^{\text{conn}} P_\alpha. \quad (100)$$

This is the  $K_5$  analogue of the scattering amplitude. It is not imported from field theory but follows from the fact that any scattering experiment has the structure: prepared motif  $\rightarrow$  finite interaction region  $\rightarrow$  detected motif.

## 24.5 What becomes the virtual particle

Nothing.

In the standard diagram for  $e^-e^- \rightarrow e^-e^-$ , one draws an internal photon line. In  $K_5$ : two Type B charged defects on the past boundary, two on the future boundary, and the internal U(1) phases rearrange between them. The correlated response of the phase network is  $G_{U(1)}(x, y) = \langle \theta_x \theta_y \rangle$ —the response kernel, not a flying object.

For nuclear scattering ( $NN \rightarrow NN$ ), the internal  $k=2$  bridge sector defines the reduced Green function in the 5-irrep:

$$\mathcal{G}_{NN}^{(5)}(E) = \langle NN, 5 | (E - H_{\text{red}}^{(5)})^{-1} | NN, 5 \rangle, \quad (101)$$

where  $H_{\text{red}}^{(5)}$  is the closure–bridge Hamiltonian from §26.9. A bound state is a pole below threshold ( $E < 2m_N$ ); a resonance is a finite-width bridge response. What perturbative language calls “pion exchange” is a pole of this response operator.

The perturbative language of virtual particles is highly useful when a compact phase integral is expanded around an ordered saddle. However, a line in a Feynman diagram is not an ontological object in  $K_5$ . Stable observed particles correspond to stable causal motifs. Internal diagram lines correspond strictly to terms in an expansion of the response kernel of the phase network. A virtual particle is not denied as a calculation; it is denied as a primitive explanatory object. This distinction is necessary to prevent replacing one physical mythology with another.

## 24.6 Worked example: elastic scattering of charged Type-B defects

To demonstrate how the  $K_5$  construction recovers familiar dynamical observables, consider the simplest elastic process: two charged Type B defects entering and leaving.

*The Gaussian ordered-phase limit.* In the ordered phase, phase mismatches are small:  $\cos \Phi \approx 1 - \Phi^2/2$ . The leading action becomes  $S \approx (\beta/2) \theta^\top H \theta$ , where  $H = B^\top B$  is the Hessian. In the presence of a boundary current  $J$  (tied to the charged defects), the action is

$$S[\theta, J] = \frac{\beta}{2} \theta^\top H \theta - J^\top \theta. \quad (102)$$

Gaussian integration over the internal phases yields the exact response:

$$\ln Z[J] = \frac{1}{2\beta} J^\top H^+ J, \quad (103)$$

where  $H^+$  is the gauge-fixed pseudoinverse.

There is no “virtual photon” in this expression. There is only  $H^+$ —the response kernel of the U(1) phase network. What perturbative language calls an internal photon line is a component of  $H^+$ .

*The long-distance limit.* At long distances, the synchronised  $K_5$  network transitions to a continuum collective limit. For the massless U(1) phase sector,  $H(q) \sim q^2$ , and the pseudoinverse scales as  $H^+(q) \sim 1/q^2$ . The connected transition kernel between two charged defects:

$$\mathcal{A}(q) = \frac{4\pi\alpha_{\text{eff}}}{q^2}. \quad (104)$$

At the bare  $K_5$  level  $\alpha^{-1} = 60$ ; at low energies  $\alpha^{-1} \approx 137$  after network dressing. The structural content is that  $K_5$  guarantees a  $1/q^2$  amplitude from the massless phase sector.

*Recovering the Rutherford limit.* Fourier transform:  $V(r) = \alpha/r$  (Coulomb potential). First Born approximation for two distinguishable nonrelativistic defects with  $|\mathbf{q}| = 2\mu v \sin(\theta/2)$ :

$$\frac{d\sigma}{d\Omega} = \left( \frac{\alpha}{4E_{\text{cm}} \sin^2(\theta/2)} \right)^2. \quad (105)$$

The chain is complete:

$K_5$  boundary kernel  $\rightarrow H^+(q) \sim 1/q^2 \rightarrow V(r) = \alpha/r \rightarrow d\sigma/d\Omega \propto 1/\sin^4(\theta/2)$ .

*Mott and Møller corrections.* The internal phase kernel does not change—it remains  $H^+(q) \sim 1/q^2$ . What changes are the external motif residues. The electron Type B defect has a fermionic carrier with a spinor projection (§17.2). In  $K_5$  language, this is the strict projection of the Type B carrier onto its long-distance fermionic external mode:  $j^\mu = \bar{u}\gamma^\mu u$ .

For two identical electrons, indistinguishability of the external motifs requires adding the exchange channel ( $q \rightarrow q' = p_1 - p_4$ ). This reproduces the standard Møller scattering structure: the  $1/q^2$  propagator from the U(1) phase response, the spinor currents from the external Type B defect residues, the exchange term from the indistinguishability of the boundary motifs.

The same construction in static scattering off a heavy source yields the Mott correction.

*Status.* The Rutherford, Mott, and Møller structures are directly recovered as the ordered long-distance limit of the  $K_5$  transition kernel, requiring no fundamental ontology of virtual intermediate particles.

## 24.7 Loop corrections and the anomalous magnetic moment

The Gaussian ordered-phase calculation above gives the tree-level response. It does not exhaust the  $K_5$  transition kernel. The full Wilson action is compact and non-linear:

$$S_W = \beta \sum_f (1 - \cos \Phi_f) = \frac{\beta}{2} \sum_f \Phi_f^2 - \frac{\beta}{24} \sum_f \Phi_f^4 + \frac{\beta}{720} \sum_f \Phi_f^6 - \dots \quad (106)$$

Writing  $S_W = S_G + S_{\text{int}}$ , the connected boundary functional is

$$W[J] = \log Z[J] = W_0[J] - \langle S_{\text{int}} \rangle_{J,c} + \frac{1}{2} \langle S_{\text{int}}^2 \rangle_{J,c} - \dots \quad (107)$$

These connected cumulants are the  $K_5$  origin of loop corrections. In diagrammatic language they are drawn as loops, but in the  $K_5$  ontology they are not virtual particles. They are non-Gaussian response terms of the compact phase network.

Fermionic loops arise in the same way from the  $K_5$  carrier operator. After the  $10 \oplus \bar{5}$  carrier has been derived (§18.4), its internal response in a background edge phase  $\theta$  can be represented by a finite-dimensional operator  $D_{K_5}[\theta]$ . Integrating the internal carrier configurations gives

$$Z[J] = \int \mathcal{D}\theta e^{-S_W[\theta] + iJ\cdot\theta} \det D_{K_5}[\theta], \quad (108)$$

and

$$\log \det D_{K_5}[\theta] = \log \det D_0 + \sum_{n \geq 1} \frac{(-1)^{n+1}}{n} \text{Tr}[(D_0^{-1}V[\theta])^n]. \quad (109)$$

These trace terms are the  $K_5$ -native form of fermion loops.

As a concrete observable, the electron anomalous magnetic moment is extracted from the three-boundary response of a Type B electron motif with one external U(1) phase insertion. The residue-normalised vertex is

$$\Gamma_{K_5}^\mu(p', p) = Z_e^{-1} P_e \langle \mathcal{O}_e(p') J_{\text{edge}}^\mu(q) \mathcal{O}_e(p) \rangle_{\text{conn}} P_e. \quad (110)$$

At long distance this vertex decomposes as

$$\bar{u}(p') \Gamma_{K_5}^\mu u(p) = \bar{u}(p') \left[ F_1(q^2) \gamma^\mu + \frac{i\sigma^{\mu\nu} q_\nu}{2m_e} F_2(q^2) \right] u(p). \quad (111)$$

The anomalous magnetic moment is  $a_e = (g-2)/2 = F_2(0)$ . The first connected vertex cumulant gives

$$F_2(0) = \frac{\alpha_{\text{eff}}}{2\pi} + O(\alpha_{\text{eff}}^2), \quad (112)$$

the Schwinger term [15]. Higher-order QED loop coefficients are recovered as the continuum ordered-phase expansion of the same  $K_5$  boundary kernel.

The standard loop expansion is not imported as a separate theory; it is an efficient long-distance representation of the connected cumulants of the  $K_5$  response functional. Possible  $K_5$ -specific corrections are suppressed by the microscopic causal scale and by irrelevant higher Wilson operators.

This completes the transition from microscopic  $K_5$  dynamics to observable scattering and radiative corrections. The next section addresses the remaining question: how the coherent phase structure of  $K_5$  produces the probabilistic and contextual behaviour observed in quantum experiments.

## 24.8 Unitarity, transfer kernels, and reduced channels

The Wilson transfer object in  $K_5$  should not be confused with a real-time unitary operator. Each individual edge transporter is unitary:

$$g_e = e^{i\theta_e} \in U(1), \quad g_e^\dagger g_e = 1. \quad (113)$$

Therefore every oriented phase transport  $U_\gamma = \prod_{e \in \gamma} g_e$  is unitary. This is an exact microscopic statement.

The Wilson weight is a different object. It is a positive statistical weight on phase configurations,  $e^{-S_W[\theta]}$ , and defines a Euclidean transfer kernel. It is not itself the real-time unitary evolution operator. In the ordered phase the positive transfer operator reconstructs an effective Hamiltonian,  $T_W = e^{-aH_{\text{eff}}}$ , so that real-time evolution is obtained only after reconstruction as  $U_{\text{eff}}(t) = e^{-iH_{\text{eff}}t}$ .

The boundary response kernel (§24) is a coherent phase sum:

$$K_{\beta\alpha} = \int_{\Theta(\mathcal{G}_*; \alpha, \beta)} \mathcal{D}\theta e^{-S_W[\theta]} U_{\beta\alpha}[\theta]. \quad (114)$$

It is not generally unitary ( $K^\dagger K \neq I$ ). It is a transition amplitude, not the full microscopic unitary evolution. Probabilities are obtained by gluing the oriented kernel to its reversed orientation:

$$P(\beta|\alpha) = \frac{K_{\beta\alpha} \overline{K_{\beta\alpha}}}{\sum_{\beta'} K_{\beta'\alpha} \overline{K_{\beta'\alpha}}}. \quad (115)$$

This normalisation guarantees  $\sum_\beta P(\beta|\alpha) = 1$ , but it is not the same statement as  $K^\dagger K = I$ .

A completely positive trace-preserving (CPTP) channel appears after coarse-graining over internal phase configurations that are not resolved by the observer. If  $p_\omega = e^{-S_W(\omega)} / \sum_{\omega'} e^{-S_W(\omega')}$  and  $K_\omega = \sqrt{p_\omega} U_\omega$ , then

$$\mathcal{E}(\rho) = \sum_\omega K_\omega \rho K_\omega^\dagger, \quad \sum_\omega K_\omega^\dagger K_\omega = I. \quad (116)$$

Thus reduced observed dynamics is CPTP, while coherent scattering amplitudes are boundary response kernels. The coherent kernel  $K$  and the reduced channel  $\mathcal{E}$  are different projections of the same  $K_5$  phase ensemble.

The usual unitary S-matrix is recovered in the infrared stable-motif sector. Gapped and decohering channels are exponentially suppressed at long distance. The surviving pole/residue subspace admits an effective Hamiltonian description. On this asymptotic subspace the reconstructed scattering matrix satisfies

$$S^\dagger S = I \quad (117)$$

up to corrections from omitted gapped sectors and irrelevant microscopic operators. Unitarity is an infrared property of the stable motif sector, whereas the microscopic Wilson transfer kernel is a positive causal response object.

For a sealed causal region (black hole, §22.4.8), the exterior map is reduced and therefore CPTP-like rather than unitary on the accessible subsystem. The full boundary-history description need not destroy information, but the explicit evaporation map is not yet derived.

## 24.9 Distant motifs and the multi-motif limit

In the  $K_5$  framework there is no separate many-body postulate. There is no fundamental empty container equipped with a Fock space of creation and annihilation operators. The network is one causal object. What standard language calls a multi-particle state is, in  $K_5$ , a single boundary phase configuration containing several localised stable motifs.

The behaviour of separated motifs is governed by the same transfer operator that governs a single motif. Approximate independence at large separation is therefore not an additional axiom. It follows from the gap structure of the transfer across shared  $K_4$  seams. Modes with nonzero transfer gap are exponentially suppressed across  $d$  seams. In the elementary seam spectrum, subleading modes are suppressed by factors of order  $(1/4)^d$ . The only correlations that survive to macroscopic distances are carried by gapless sectors: coherent  $U(1)$  phase non-equivalence response and the collective gravitational response. These produce the observed long-range power-law tails rather than exact independence.

Thus the usual cluster decomposition is replaced by a more precise  $K_5$  statement: distant motifs decouple up to universal gapless response kernels. For gapped sectors the decoupling is exponential. For massless sectors the residual correlation is the appropriate long-distance Green function.

Particle identity is also not a separate postulate. Two motifs with the same quantum numbers are two local realisations of the same causal pattern. Their exchange is a network automorphism, up to the projective phase fixed by the binary-cover representation of the spatial-channel algebra (§17.4). The usual multi-particle language is therefore an infrared bookkeeping device for patterns in one network, not an additional ontology.

## 24.10 Optical theorem and crossing

The optical theorem is not an additional postulate of the  $K_5$  framework. It is the infrared consequence of the reconstructed unitary S-matrix on the stable motif subspace (§24.8). Once the boundary response kernel is projected to asymptotic motif poles and gapped channels are removed, the resulting S-matrix satisfies  $S^\dagger S = I$ . Writing  $S = I + iT$  gives  $-i(T - T^\dagger) = T^\dagger T$ , and therefore

$$2 \operatorname{Im} \mathcal{A}_{\alpha \rightarrow \alpha} = \sum_{\beta} |\mathcal{A}_{\alpha \rightarrow \beta}|^2. \quad (118)$$

The optical theorem is the probability-conservation statement of the stable-motif sector.

Crossing symmetry has a distinct status. In  $K_5$ , antiparticles are orientation-reversed motifs: reversing the edge transporter sends  $g_e = e^{i\theta_e} \mapsto g_e^{-1} = e^{-i\theta_e}$ . Moving a boundary motif from the future boundary to the past boundary is therefore represented by boundary-orientation reversal. This gives the structural origin of crossing. A complete crossing theorem additionally requires the analyticity of the boundary response kernel in the external transfer eigenvalues. Crossing is therefore structurally expected from orientation reversal but is not yet an independent theorem of the present manuscript.

# 25 Quantum foundations

## 25.1 Born rule from $K_5$ gluing

The Born rule has been derived in §24.3 from oriented phase gluing. The correct  $K_5$  contribution of a phase configuration  $\omega$  is  $e^{-S_W[\omega]} U_{\beta\alpha}[\omega]$ : a real positive Wilson weight times a unitary edge

transporter. The boundary response kernel is the coherent sum

$$K_{\beta\alpha} = \int \mathcal{D}\theta e^{-S_W[\theta]} U_{\beta\alpha}[\theta]. \quad (119)$$

Observable probabilities are obtained by gluing oriented and reverse-oriented kernels:

$$P(\beta|\alpha) = \frac{K_{\beta\alpha} \overline{K_{\beta\alpha}}}{\sum_{\beta'} K_{\beta'\alpha} \overline{K_{\beta'\alpha}}}. \quad (120)$$

Three requirements on  $P$ : (i) positivity ( $P \geq 0$ ); (ii) gauge invariance (independence from local phase redefinition); (iii) additivity for decoherent alternatives. The  $|K|^2$  form is the unique function satisfying all three.

For two coherent alternatives:  $P = |K_1 + K_2|^2 = |K_1|^2 + |K_2|^2 + 2 \operatorname{Re}(K_1 \overline{K_2})$ . The interference term  $2 \operatorname{Re}(K_1 \overline{K_2})$  measures the coherence of two admissible gluings. Absolute phase is unobservable (gauge); relative phase is observable (interference).

## 25.2 Bell inequality violation [27]

Within  $A_4$ , the subgroups  $V_4$  (spatial half-turn kernel) and  $\mathbb{Z}_3$  (channel cycling) do not commute. Observers in different causal positions are forced to use incompatible bases to describe the same system. This reproduces quantum correlations not through nonlocality, but through the incompatibility of local descriptions (contextuality).

*Contextual compatibility of one causal motif.* In  $K_5$  the measurement settings and the hidden variables live on the same graph: both are properties of one edge-phase configuration  $\{\theta_k\}$ . The correlation between choice of basis and state of the system is a topological fact of the shared causal motif, not a conspiracy.

*Gaussian limit (Werner–Wolf).* In the Gaussian (free, 1-loop) approximation, the covariance matrix of chiral-sector observables has  $\Sigma_{AB} = 0$  exactly: the chiral sectors  $3$  and  $3'$  are completely independent. By the Werner–Wolf theorem [17], Gaussian quantum states satisfy all Bell inequalities.  $K_5$  at 1-loop is Gaussian, so Bell violation is impossible in the free theory. This is not a deficiency—it is the correct behaviour of a non-interacting theory.

*Why interactions turn on Bell violation.* When interactions are turned on—the anharmonic terms  $\cos \Phi \neq 1 - \Phi^2/2$ —the chiral sectors mix. Anharmonic terms create non-factorisable amplitudes; inter-saddle phases produce correlations that can violate the Bell bound.

$K_5$  gives a causal-topological reading of Bell correlations: the failure is the independent-particle answer-table factorisation, not causality. A full quantitative Bell-test probability model (computing  $|S|_{\text{CHSH}}$  from the  $K_5$  response kernel) is not claimed in the present manuscript.

## 25.3 Decoherence [28]

The environment adds edges to the graph, entangling the system with its surroundings. The visibility of interference between two paths:

$$\langle V \rangle = \exp(-M/137), \quad (121)$$

where  $M$  is the number of environment edges coupled to the system. For a macroscopic object in air,  $M \sim 10^{20}$ , giving  $V = 0$ . Decoherence is not a postulate; it is a consequence of graph growth.

# 26 Vacuum and baryons

## 26.1 Vacuum as a medium

The  $K_5$  network in the ordered phase is not empty space with fields laid on top. It is a medium with concrete measurable properties, analogous to superfluid  $^3\text{He}$ :

Medium property	$K_5$ analogue	Physical content
Elastic modulus	$\beta$ (lattice coupling)	$\beta = 5\text{--}10$ on LCP
Order parameter	$\langle e^{i\varphi} \rangle \neq 0$	OP $> 0.87$ at $\beta \geq 1$
Stiffness	$\Delta S/\Delta^2$	$\approx 2500$
Defects	particles (Type B = electron)	
Collective modes (IR)	gravity: $G = \sqrt{3}/(32\pi\beta)$	gapless
Local excitations (UV)	gauge bosons: mass gap = 5	gapped
Emergent scale	$\ell_P \sim a/\sqrt{\beta}$	

The analogy to  $^3\text{He}$  is structural, not decorative. Phonons = collective modes. Rotons = local excitations. Vortices = topological defects. Superfluidity = long-range order. The macroscopic vacuum is not an empty stage; it is exactly this synchronised medium of  $K_5$  cells.

Three consequences: the cosmological constant is not vacuum energy but the finite-size response of the medium ( $\Lambda_{\text{local}} = 0$  from  $|E| = |F|$ ); gravity is the elasticity of the medium, not a separate force; dark energy is the network response modulus at cosmological scale.

Spontaneous symmetry breaking:  $\Gamma^2|_4 = -3/20$  (exact Gaussian LO, confirmed by MC to  $-0.1534 \pm 0.0007$ ). The  $A_5 \rightarrow A_4$  breaking is energetically preferred.

## 26.2 Baryonic layer

From  $K_5$  fermion content  $\bar{5} \oplus 10$  and confined  $\text{SU}(3)$ : the next composite level is the colour-singlet 3-quark baryon:

$$B^{f_1 f_2 f_3}(x) = \varepsilon_{abc} q^{a, f_1}(x) q^{b, f_2}(x) q^{c, f_3}(x). \quad (122)$$

Not a new postulate—follows from  $3 \otimes 3 \otimes 3 = 1 \oplus 8 \oplus 8 \oplus 10$ , where the unique singlet =  $\varepsilon$ -contraction.

For quarks at different lattice sites  $x_1, x_2, x_3$  with junction  $x_J$ , the gauge-covariant nonlocal operator:

$$B(x_1, x_2, x_3; x_J) = \varepsilon_{abc} [U(x_J, x_1)q(x_1)]^a [U(x_J, x_2)q(x_2)]^b [U(x_J, x_3)q(x_3)]^c, \quad (123)$$

where  $U(x_J, x_i)$  is the induced  $\text{SU}(3)$  transporter from the colour sector.

This has a consequence for gravity. The physical source  $T_{\mu\nu}$  must be baryonic—a colour singlet with finite size, exclusion, and an equation of state. The chain  $K_5 \rightarrow \bar{5} \oplus 10 \rightarrow \text{SU}(3)$  confinement  $\rightarrow$  baryon singlet  $\rightarrow$  EOS  $\rightarrow T_{\mu\nu}^{\text{matter}}$  is the bridge between microscopic  $K_5$  and macroscopic gravity.

## 26.3 Proton core mass: causal closure

The electron and the proton share the same mass law:  $m = M_{\text{P1}} \cdot \exp(-S_{\text{eff}})$ . What distinguishes them is not their Lagrangian but their topology.

The electron is a single-stream defect. Its topological cost is  $S_0(e) = |A_5| - |E(K_5)| + 1 = 51$ . The proton is a three-stream causal closure—three colour streams locked together by the  $\varepsilon$ -contraction. Its cost is

$$S_0(p) = |A_5| - |E(K_5)| - |F(K_5)| + 1 = 60 - 10 - 10 + 1 = 41. \quad (124)$$

The difference  $S_0(e) - S_0(p) = |F(K_5)| = 10$  has a sharp physical meaning. The 10 face holonomies that penalise the electron (as Bianchi constraints, each adding to the cost) become bonds that stabilise the proton (the  $\varepsilon$ -contraction turns a penalty into a coupling). Colour closure converts constraints into connections.

**Theorem 26.1** (Closure Hessian). *Let  $B = \partial_2$  be the face-to-edge boundary operator ( $10 \times 10$ ), and  $T = \{T_1, T_2, T_3\}$  any three of the five tetrahedra of  $K_5$ . Define  $|D_f|$  as the number of tetrahedra from  $T$  containing face  $f$ . Then*

$$H_T = B^\top \text{diag}(|D_f|) B \quad (125)$$

has spectrum  $\{0^4, 2, 5, 5, 7, 7, 10\}$ , independently of which three tetrahedra are chosen.

The proof follows the baryon construction step by step. Each stream contributes  $S_t = (\beta/2) \theta^\top B^\top \Pi_t B \theta$ , where  $\Pi_t$  projects onto the 4 faces of tetrahedron  $T_t$ . The  $\varepsilon$ -contraction couples streams through shared edge phases, and the actions add:  $S_{\text{baryon}} = (\beta/2) \theta^\top B^\top (\sum_t \Pi_t) B \theta$ . Since  $\sum_t \Pi_t = \text{diag}(|D_f|)$ , the result follows. The spectrum is  $S_5$ -invariant because  $S_5$  acts transitively on all  $\binom{5}{3} = 10$  triples.

The analytic structure is revealing. The mean eigenvalue on the physical subspace is 6. The shifts from this mean— $\{-4, -1, -1, +1, +1, +4\}$ —form a pattern with  $\mathbb{Z}_2 \times S_3$  symmetry. The softest mode ( $\lambda = 2$ ) transforms as the sign representation of  $S_3$ : it literally *is*  $\varepsilon_{abc}$ , the baryonic channel. The stiffest ( $\lambda = 10$ ) is the backbone closure.

The Gaussian closure cost:

$$\Delta S_{\text{clo}} = \frac{1}{2} \ln(\det' H_T / \det' H_{\text{tet}}) = \frac{1}{2} \ln(24500/64) = 2.97. \quad (126)$$

This determinant ratio is the total action cost of completing an open colour channel into the first full baryonic closure. It should not be read as a quark pole mass. It is the closure action of a rank-incomplete channel when the  $k = 1$  strand is completed through the  $k = 2$  bridge into the  $k = 3$  singlet closure.

The proton mass:

$$S_{\text{eff}}(p) = 41 + 2.97 = 43.97, \quad m_p^{\text{core}} = M_{\text{Pl}} \cdot e^{-43.97} \approx 979 \text{ MeV}. \quad (127)$$

The observed value is 938 MeV—an agreement to 4.4%. Because the mass law is  $m = M_{\text{Pl}} e^{-S}$ , the next calculation has a precise benchmark rather than a vague “network dressing” target:

$$\Delta S_p^{\text{NLO}} = \ln\left(\frac{m_p^{\text{core}}}{m_p^{\text{pole}}}\right) = \ln\left(\frac{979}{938}\right) \simeq 4.28 \times 10^{-2}. \quad (128)$$

Thus a full multi-cell closure dressing should increase the proton effective action by about 0.043, shifting the leading core value down to the observed pole mass. If the closed NLO operator gives a correction of a parametrically different size or sign, the proton sector is under direct pressure.

	Electron	Proton
Topology	single-stream defect	three-stream closure
$S_0$	$ A_5  -  E  + 1 = 51$	$ A_5  -  E  -  F  + 1 = 41$
Role of faces	constraints (penalty)	bonds (coupling)
$\Delta S$	0.47	2.97
$S_{\text{eff}}$	51.53	43.97
$m$	0.52 MeV	979 MeV (core)

## 26.4 $k$ -tet causal closure hierarchy

For any subset  $T \subseteq \{5 \text{ tetrahedra of } K_5\}$ ,  $|T| = k$ , the  $k$ -tet closure Hessian is  $H_T = B^\top \text{diag}(|D_f(T)|) B$ .

**Theorem 26.2** ( $k$ -tet hierarchy). *spec( $H_T$ ) depends only on  $k = |T|$ , not on the specific choice. For each  $k$  there exists a unique  $S_5$ -invariant spectral passport.*

$k$	Physical spectrum	$\det'$	Phys. rank	Object
1	{4, 4, 4, 0, 0, 0}	$4^3 = 64$	3/6	open colour strand
2	{3, 3, 5, 5, 8, 0}	$3^2 \cdot 5^2 \cdot 8 = 1800$	5/6	meson-like bridge
3	{2, 5, 5, 7, 7, 10}	$2 \cdot 5^2 \cdot 7^2 \cdot 10 = 24500$	6/6	baryon closure
4	{6, 6, 6, 10, 10, 10}	$6^3 \cdot 10^3 = 216000$	6/6	BH overclosure seed
5	{10, 10, 10, 10, 10, 10}	$10^6$	6/6	saturated vacuum

Verified: all  $\binom{5}{k}$  choices for each  $k$  give the same spectrum.

This table should be read as a finite periodic table of causal motifs. The sequence is not a list of five particle types:  $k = 1$  is an open colour channel,  $k = 2$  is a bridge sector,  $k = 3$  is the first closed baryonic sector,  $k = 4$  is the first overclosed local sealed-cluster seed, and  $k = 5$  is the saturated vacuum. The dual pairs are  $k = 1 \leftrightarrow 4$  and  $k = 2 \leftrightarrow 3$ ; the black-hole interpretation of the  $k = 4$  entry is precisely the local theorem 22.6, while the horizon and entropy statements require the Layer-4 sealed-cluster construction of §22.4.

**Channel-completion ladder.** The first three entries form a single completion ladder,

$$k = 1 \longrightarrow k = 2 \longrightarrow k = 3,$$

namely

$$\text{open colour channel} \longrightarrow \text{bridge} \longrightarrow \text{baryon closure}.$$

This is the actual  $K_5$  gluing lattice relevant for colour dynamics. It is not a flat integer lattice of mass-squared values. The matrices  $H_1, H_2, H_3$  are the same weighted face-stiffness operator with different numbers of tetrahedral streams. A rank-incomplete quark channel is therefore neither an asymptotic particle nor a fiction; it is the open charged colour channel whose completion cost is read through the bridge and the first full baryonic closure.

**Corollary 26.3** (Trace).  $\text{Tr}(H_k) = 12k$ . Each tetrahedron contributes 4 faces, each face contributes 3 to the trace through  $B^\top WB$ . Total:  $4 \times 3 = 12$  per tetrahedron. Each new stream adds exactly one portion of stiffness.

**Corollary 26.4** (Duality  $k \leftrightarrow 5 - k$ ). Each face of  $K_5$  belongs to exactly 2 of 5 tetrahedra:  $|D_f(T)| + |D_f(T^c)| = 2$ . Therefore  $H_k + H_{5-k} = 10 \cdot I_6$  on the physical subspace. Spectral duality:  $\lambda(5 - k) = 10 - \lambda(k)$ . Pairs:  $k=1 \leftrightarrow k=4$ ,  $k=2 \leftrightarrow k=3$ ;  $k=5$  is the self-dual point.

*Confinement as rank completeness.* At  $k = 1$ , only 3 of 6 physical modes are active—a quark cannot exist in isolation because its colour strand has incomplete rank. This is not dynamical trapping. It is a statement about the algebra: a  $k=1$  open strand simply does not close the  $B^\top WB$  kernel. At  $k = 2$ , 5 of 6 modes are active—almost closed (a meson-like bridge, with one zero mode that becomes the pion). At  $k = 3$ , all 6 physical modes are active—the first fully closed object.

**Corollary 26.5.** Single-quark observables are a category error. A quark is a  $k=1$  open strand with rank 3/6. Physical observables begin at  $k=3$ : gauge-invariant baryon correlators, meson-baryon splittings, the static three-quark potential. Measuring a “quark mass” is an attempt to treat an incomplete colour strand as a closed object. In the SM, the quark is a fundamental field with its own mass  $m_q$  and confinement is dynamical. In  $K_5$ , confinement is kinematic (rank completeness), and  $m_q$  as an isolated number is undefined. Quark mass ratios appear only at the hadronic level through closure and bridge spectra.

**Map-type guardrail.** The  $k$ -tet hierarchy is a closure-sector decomposition, not a projection of a proton. The  $k = 1$  strand,  $k = 2$  bridge,  $k = 3$  baryon closure,  $k = 4$  overclosure, and  $k = 5$  saturated vacuum are rank/closure sectors of the same  $K_5$  closure algebra. In particular, a quark channel is not a little projected proton constituent; it is a rank-incomplete open strand that becomes physical only inside a closed motif.

A high-temperature quark–gluon plasma is therefore not, in this language, a phase in which isolated quark poles become valid asymptotic objects. It is a change of closure ensemble and screening regime inside the same rank-completeness rule. This is a testable ontological distinction: deconfinement may weaken long-range hadronic binding, but it should not produce an isolated single-quark external state.

## 26.5 Higgs mass from closure inverse stiffness

**Theorem 26.6** (Negative stiffness of the symmetric point).  $\Gamma^{(2)}|_4 = -3/20 \cdot I_4$ . *The  $A_5$ -symmetric configuration is a saddle point;  $A_5 \rightarrow A_4$  breaking is energetically preferred. MC confirmed (500k configs):  $-0.1534 \pm 0.0007$  vs LO  $-0.1500$ .*

**Theorem 26.7** (Gauge does not stabilise).  $\lambda_{gauge} < 0$ . *Verified: Gaussian ( $\Gamma^{(4)}(0) = -0.045$ ) and full Wilson MC on 2-cell chain with gluing. The gauge sector creates a breaking direction but not a stable minimum.*

**Theorem 26.8** (Radial-only fermion stabilisation). *Temporal gluing through  $\text{tet}_0 = \{1, 2, 3, 4\}$  distinguishes radial and Goldstone directions. Net perturbation on gluing faces: radial =  $4 \times 3/\sqrt{20} \neq 0$ ; Goldstone = 0 (exact). The fermion determinant  $\det(D)$  couples only to the radial mode. Three Goldstone directions remain exactly massless. The reason: Goldstone modes permute  $v_1 \dots v_4$  symmetrically; gluing through  $\text{tet}_0$  sees them identically; the sum of perturbations vanishes.*

*Collective quartic from minimal closure.* The effective quartic coupling is determined by the inverse stiffness of the minimal fully closed  $k=3$  causal object:

$$\lambda_{\text{eff}} = \frac{1}{10} \text{Tr}'(H_{k=3}^{-1}) = \frac{9}{70}. \quad (129)$$

Here  $H_{k=3}$  has spectrum  $\{2, 5, 5, 7, 7, 10\}$ .  $\text{Tr}'(H^{-1}) = 1/2 + 2/5 + 2/7 + 1/10 = 9/7$ . Per edge:  $9/70$ —an exact rational  $K_5$  number from the  $k=3$  closure spectrum.

**Corollary 26.9** (Higgs mass).  $\lambda_{\text{eff}}/\mu^2 = (9/70)/(3/20) = 6/7$ .

$$m_H/v = \sqrt{2\lambda_{\text{eff}}} = \sqrt{9/35} = \frac{3}{\sqrt{35}} = 0.5071. \quad (130)$$

At  $v = 246 \text{ GeV}$ :  $m_H = 124.9 \text{ GeV}$  (experiment:  $125.1 \pm 0.1$ ,  $\Delta = 0.2\%$ ). Zero free parameters.

The Higgs in  $K_5$  is not the source of fermion masses (§19.12) and not a separate fundamental scalar. It is the radial mode whose quartic coupling is fixed by the inverse stiffness of the minimal baryonic closure. Three sectors are linked by one formula: symmetry breaking  $A_5 \rightarrow A_4$  ( $\mu^2 = 3/20$ ), baryonic closure  $k=3$  ( $\text{Tr}' H^{-1}/10 = 9/70$ ), and the fermion determinant (radial-only stabilisation). All three are necessary for  $m_H$ ; none is sufficient alone.

## 26.6 Neutron–proton splitting

Proton and neutron are the same  $k=3$  closure with different isospin filling. The baryonic space factorises:  $\mathcal{H}_{\text{baryon}} = \mathcal{H}_Y \otimes \mathcal{H}_{\text{iso}}$ , where  $\mathcal{H}_Y$  is the common colour-singlet closure (same for  $p$  and  $n$ ) and  $\mathcal{H}_{\text{iso}}$  is the isospin doublet. The common closure Hamiltonian  $H_Y \otimes I$  cancels in  $m_n - m_p$ .

The LO splitting operator is unique. Requirements: (i)  $T_3$ -odd (distinguishes  $p$  from  $n$ ); (ii) lives on  $k=3$  closure; (iii)  $K_5$ -native (no import); (iv) leading order (lowest dimension). On  $k=3$  closure, the unique gauge scalar on the physical 1-form sector is  $K_{\text{phys}} = B^\top B|_{\text{phys}} = 5I$  (bare eigenvalue, verified). The unique isospin generator is  $T_3^{\text{tot}}$ . The unique EW mass scale is  $m_e$  (minimal charged defect). The product is therefore unique:

$$\delta H_{np} = -\frac{m_e}{2} \cdot K_{\text{phys}} \otimes T_3^{\text{tot}} = -\frac{5}{2} m_e \cdot T_3^{\text{tot}}. \quad (131)$$

With  $T_3^{\text{tot}}|p\rangle = +\frac{1}{2}|p\rangle$  and  $T_3^{\text{tot}}|n\rangle = -\frac{1}{2}|n\rangle$ :  $\delta m_p = -(5/4)m_e$ ,  $\delta m_n = +(5/4)m_e$ .

$$m_n - m_p = \frac{5}{2} m_e = 1.278 \text{ MeV} \quad (\text{experiment: } 1.293, \Delta = 1.2\%). \quad (132)$$

Quantity	$K_5$	Experiment
$m_n - m_p$	$(5/2) m_e = 1.278 \text{ MeV}$	1.293 MeV (1.2%)
$(m_n - m_p)/m_p$	$(5/2) \exp(-\Delta S)$	0.001378 (0.3%)

Why bare  $K = 5$ , not dressed  $K = 6$ : the closure eigenvalues  $\{2, 5, 5, 7, 7, 10\}$  (mean 6) describe the colour incidence of the object  $Y$ . But  $T_3$  acts within the isospin doublet—orthogonal to colour. Closure dressing lives in the common  $M_{\text{clo}}$ , not in the coefficient of  $T_3$ . Check:  $K = 5$  gives 1.278 MeV (1.2% off);  $K = 6$  gives 1.533 MeV (18.5% off).

Strong/EM separation is not required. In the SM the splitting decomposes into QCD (+2.52 MeV) and QED (−1.00 MeV). In  $K_5$  there is one gauge coupling  $\beta$ , one bare stiffness 5, one operator. The separation into “strong” and “EM” is SM language, not  $K_5$ -native.  $K_5$  gives the full answer directly.

The formula connects three independent  $K_5$  objects: the electron (defect), the proton (closure), and the neutron–proton splitting (isospin on closure). Zero free parameters.

## 26.7 Pion resonance law on closure background

The pion is not a primitive defect. Like the proton, it is not a Type B topological defect and not a new particle. It is the lightest isotriplet resonance of a  $k=2$  bridge on a  $k=3$  baryonic closure background. The pion sector requires no new mechanism: it follows from the  $k$ -tet hierarchy (§26.4) and protonic closure (§26.3).

*Three exact  $K_5$  numbers.*

(1) Zero-mode lift from closure background. The  $k=2$  bridge (spectrum  $\{3, 3, 5, 5, 8, 0\}$ ) has one zero mode corresponding to the missing third stream. In isolation this is a flat direction; in the presence of baryonic closure it acquires a mass. The spectral shift:

$$\Delta H = H_{k=3} - H_{k=2} = \{-1, +2, +2, +2, -1, +10\}. \quad (133)$$

Average shift on the zero mode:  $\langle 0^{k=2} | \Delta H | 0^{k=2} \rangle = (-1 + 2 + 2 + 2 - 1 + 10)/6 = 14/6$ . But the zero mode has overlap only with the physical subspace of  $k=3$  (rank 6): projection weight  $10/14 = 5/7$ . Effective lift:

$$\langle 0^{k=2} | H_{k=3} | 0^{k=2} \rangle_{\text{phys}} = \frac{14}{6} \times \frac{5}{7} = \frac{10}{3}. \quad (134)$$

An exact rational  $K_5$  number: 10 = faces of  $K_5$ , 3 = colour streams.

(2) Average closure stiffness:  $\bar{\lambda}_{k=3} = 36/6 = 6$ .

(3) Bare gauge stiffness:  $K_{\text{phys}} = 5$  (§19.10).

*Pion mass ratio.*

$$\frac{m_\pi^2}{m_p^2} = \frac{\text{zero-mode lift}}{\bar{\lambda}_{k=3} \times K_{\text{phys}}^2} = \frac{10/3}{6 \times 25} = \frac{1}{45} = \frac{1}{9 \times 5}, \quad (135)$$

where  $6 \times 25 = \bar{\lambda} \times K^2$  (dimensional normalisation of baryonic closure through the square of gauge stiffness).

$$\frac{m_\pi}{m_p} = \frac{1}{3\sqrt{5}}, \quad m_{\pi^\pm} = \frac{m_p}{3\sqrt{5}} = 139.9 \text{ MeV}. \quad (136)$$

Quantity	$K_5$	Experiment	Agreement
$m_{\pi^\pm}$	$m_p/(3\sqrt{5}) = 139.9 \text{ MeV}$	139.57 MeV	0.2%
$(m_p/m_\pi)^2$	45	45.19	0.4%

Structure  $45 = 9 \times 5$ :  $9 = 3^2$  (colours squared),  $5 =$  bare gauge eigenvalue. This is not GMOR ( $m_\pi^2 \propto f_\pi m_q$ ), which is SM phenomenology. Here  $m_\pi^2$  is fixed by the closure structure without reference to quark masses or the chiral condensate. The closure background converts the bridge zero mode into a massive mode; the formula is an exact rational function of the closure spectrum. Zero free parameters.

**Open-channel centroid corollary.** Combining the pion law  $m_p = 3\sqrt{5} m_\pi$  with the open-channel centroid theorem of §16.3 gives

$$M_{\text{ch}}^2 = \frac{m_p^2}{3} = 15m_\pi^2. \quad (137)$$

This is a statement about the norm of one open coloured channel inside a three-channel baryonic closure. It is not an isolated quark mass and it is not a point on a flat mass-squared lattice.

**$\rho$  vector-lift candidate.** The lightest electromagnetic vector lift of the  $k = 2$  bridge is the  $\rho$  channel. The structural candidate relation is

$$m_\rho^2 = (1 + N_{30})m_\pi^2 = 31m_\pi^2, \quad N_{30} = \text{Tr}'(d_2^\top d_2) = 30. \quad (138)$$

This gives  $m_\rho \simeq 777 \text{ MeV}$  from the charged pion scale, within about 0.2–0.3% of the observed  $\rho(770)$  mass. Its status is B+/A–: the number  $31 = 1 + N_{30}$  has a  $K_5$  stiffness interpretation, but the full  $J^{PC} = 1^{--}$  vector-lift operator remains to be written.

**No flat integer mass-squared lattice.** The relations above should not be generalised into a post-hoc integer lattice  $m^2/m_\pi^2 \in \mathbb{Z}$  for arbitrary light mesons. The  $K_5$  lattice is the gluing/completion hierarchy of  $H_k$  and the sliding-window seam structure, not a flat numerical axis. Approximate integer fits for  $\omega, K^*, \phi$  are discarded unless an explicit  $K_5$  vector/flavour lift operator generates them.

## 26.8 Proton closure-radius law

The proton in  $K_5$  is the minimal fully closed causal object. Its natural size is the internal length scale of the  $k=3$  closure.

Closure spectrum  $\{2, 5, 5, 7, 7, 10\}$ : shifts from mean 6 are  $\{-4, -1, -1, +1, +1, +4\}$ . Spectral half-gap:  $\mu_{\text{max}} = 4$ .

Candidate law:

$$m_p r_p / (\hbar c) = \mu_{\text{max}} = 4, \quad r_p = 4\hbar c / m_p = 0.8412 \text{ fm}. \quad (139)$$

Measurement	$r_p$ (fm)	$m_p r_p / (\hbar c)$	Agreement
$K_5$ prediction	0.8412	4 (exact)	—
Muonic hydrogen (2010, 2013)	$0.8409 \pm 0.0004$	3.998	0.04%
PRad (2019, $e$ scattering)	$0.831 \pm 0.014$	3.95	1.2%
CODATA 2018	$0.8414 \pm 0.0019$	4.001	0.02%

$K_5$  supports the muonic value. The proton radius puzzle (the pre-2019 disagreement between electron and muonic measurements, now converging [13] to  $\sim 0.84$  fm) is consistent with the structural prediction  $m_p r_p / \hbar c = 4$ .

4 = the maximum eigenvalue shift of the closure Hessian from the mean—a measure of the maximum spectral deviation of the closed object. The proton is a causal object whose size literally equals the “width” of its closure spectrum in units of the mean. Not a string model, not an MIT bag, not a constituent quark picture—pure spectral geometry of  $K_5$ .

*Connection to form factor.*  $r_p^2 = -6 dG_E(q^2)/dq^2|_{q^2=0}$ . The  $K_5$ -derived  $r_p^2$  determines the slope of  $G_E$  through convolution with the  $k=3$  kernel. The full form factor at larger  $q^2$  requires a multi-pole fit not included in the minimal closure picture. Derivation of the form factor slope directly from the  $k=3$  closure kernel remains open.

## 26.9 Nuclear force from closure–bridge dynamics

**(R1) Single-closure representation.** The closure mode transforms under the stabiliser  $S_3 \times S_2$  of the  $T_3$ -subset as  $\text{sign}(S_3) \otimes \text{triv}(S_2)$  ( $S_3$  on excluded vertices,  $S_2$  on common). Frobenius induction:

$$10_{\text{closure}} = \text{Ind}_{S_3 \times S_2}^{S_5} (\text{sign}_{S_3} \otimes \text{triv}_{S_2}) = \mathbf{6} \oplus \mathbf{4}'. \quad (140)$$

Verified by character orthogonality:  $\chi_{\text{ind}} = (10, -2, -2, 1, 1, 0, 0)$  gives  $\langle \chi_{\text{ind}}, \chi_6 \rangle = \langle \chi_{\text{ind}}, \chi_{4'} \rangle = 1$ , all others = 0. Dimension check:  $6+4 = 10 \checkmark$ . Edge permutations must be signed (orientation flip upon vertex reordering) for  $H_T$  to commute with the  $S_5$ -action.

**(R2) Two-nucleon orientation space.** Tensor square:  $10_{\text{closure}} \otimes 10_{\text{closure}} = 2 \cdot \mathbf{1} + 4 \cdot \mathbf{4} + 5 \cdot \mathbf{5} + 4 \cdot \mathbf{6} + 4 \cdot \mathbf{5}' + 3 \cdot \mathbf{4}' + \mathbf{1}'$ . Dimension check:  $2 + 16 + 25 + 24 + 20 + 12 + 1 = 100 \checkmark$ .

The attractive channel is the 5-irrep of  $S_5$  (5 copies of the dim-5 irrep in the two-NN orientation space). Clarification:  $\lambda_-^{(5)}(d)$  is the lowest eigenvalue of the  $5 \times 5$  matrix acting on this multiplicity space, not a single eigenvalue of the 5-dimensional irrep itself.

**(R3) Closure–bridge vertex overlap.** For closure at  $T_3$  and bridge at  $T_2$  on one  $K_5$  cell:  $|\langle \text{closure}_\alpha | \text{bridge}_\beta \rangle| = \sqrt{5}/3$  on 44 non-zero entries of the  $10 \times 10$  vertex matrix. Frobenius norm:  $\|V\|_F^2 = 44 \times 5/9 = 220/9$  (exact rational, verified to  $3 \times 10^{-16}$ ). A closed-form derivation through Schur intertwiner normalisation is not yet complete.

*Structural selection rule.*  $V_{\text{direct}}^{(5)}(d) = 0$  identically. Direct gauge exchange in the attractive 5-irrep structurally vanishes: closure in the 5-irrep corresponds to antisymmetric colour modes, and the symmetric gauge propagator does not couple to antisymmetric sources. Deuteron binding must proceed exclusively through bridge-mediated  $NN \leftrightarrow \pi$  mixing.

*Exact coupling constants.*  $g_{\pi NN} = 10/3$ ,  $V_\pi$  rational. Forced Yukawa anchor:  $\kappa = m_\pi/m_p = 1/(3\sqrt{5}) \approx 0.14907$  (exact  $K_5$ ). Tail-test on data:  $\kappa_{\text{fit}} = 0.148$ —agreement 0.7%.

Nuclear force = spectral reorganisation of closure–bridge mixing. Pion exchange is the effective description of this reorganisation at large distances.

### 26.9.1 Distance dependence and emergent Yukawa

The attractive eigenvalue decays with inter-baryon separation:

$d$ (cells)	$\lambda_-$	
1	−0.160	dominant
2	−0.078	
3	−0.041	exponential decay visible
4	−0.024	
5	−0.015	approaching zero
6	−0.0094	

The decay is consistent with the Yukawa form  $\lambda_-(d) \approx A \cdot \exp(-\kappa d)/d$ . From the ratio  $\lambda(d=4)/\lambda(d=1) = 0.024/0.16 = 0.15 = \exp(-3\kappa)$ , giving  $\kappa = 0.63$  in lattice units. Calibrating with 1 cell =  $r_p = 0.841$  fm (§26.8):

$$\kappa_{\text{phys}} = 0.63 \times \frac{m_p}{4} \approx 148 \text{ MeV}. \quad (141)$$

Physical pion mass:  $m_\pi = 139.6$  MeV. Agreement 6%. This is the first appearance of the pion in nuclear force as an emergent exchange—not a hypothesised boson, but the solution of the closure–bridge eigenvalue problem.

### 26.9.2 Why bridge admixture binds

The bare two-baryon kernel is repulsive: two closures conflict because each demands full rank, and phase overlap creates interference. A bridge ( $k=2$ , rank 5/6) has one fewer mode and reduces phase frustration. The observed bridge component in reduced numerical ground states is an off-diagonal channel component, not a scalar dilution factor multiplying a one-channel Yukawa potential.

Exact bridge coupling constants from closure spectra:  $\det'(2\text{-cell closure})/\det'(1\text{-cell}) = 1445/49$ ; pion–nucleon coupling:  $g_{\pi NN} = 10/3$ ; pion coupling to 3-body amplitudes:  $D_\pi^{(1)} = 11/30$ ,  $D_\pi^{(2)} = 11/70$ ; pion potential coefficients:  $V_\pi^{(1)} = -110/27$ ,  $V_\pi^{(2)} = -110/63$ . All exact rational  $K_5$  numbers.

The attractive bridge channel is the 5-irrep of  $S_5$ . Its Schur coefficient in the bridge-mediated exchange operator is

$$c_5 = -2,$$

while direct gauge exchange in the same attractive channel remains zero by the structural selection rule above. The scalar leading-order reduction of the bridge-mediated channel is therefore not the raw Yukawa coupling. It is diluted by the fraction of the  $K_5$  edge density active in the 5-irrep:

$$\frac{|c_5|}{|E(K_5)|} = \frac{2}{10} = \frac{1}{5}.$$

The dimensionless one-channel scalar Yukawa strength is then

$$\lambda_5^{\text{LO}} = \frac{2\mu}{m_\pi} \left( \frac{g_{\pi NN}^2 |c_5|}{4\pi} \right) \left( \frac{|c_5|}{|E(K_5)|} \right). \quad (142)$$

Using  $2\mu \simeq m_N$ ,  $m_\pi/m_N = 1/(3\sqrt{5})$ ,  $g_{\pi NN} = 10/3$ ,  $|c_5| = 2$ , and  $|E(K_5)| = 10$ , this becomes

$$\lambda_5^{\text{LO}} = 3\sqrt{5} \cdot \frac{25}{9\pi} \cdot 2 \cdot \frac{1}{5} = \frac{10\sqrt{5}}{3\pi} \simeq 2.37. \quad (143)$$

Solving the scalar Yukawa threshold problem with this coupling gives

$$B_d^{\text{LO}} \simeq 2.14 \text{ MeV}, \quad (144)$$

compared with the experimental  $B_d = 2.224$  MeV. The leading scalar bridge reduction is therefore low by about 0.08 MeV, or 3.7%.

This residual is the expected size of the missing tensor/D-wave correction. The scalar reduction above keeps only the isotropic  $S$ -wave part of the 5-irrep bridge exchange. The physical deuteron contains a small  $D$ -wave component generated by the tensor part of pion exchange. In the  $K_5$  description this tensor contribution is the next Schur component of the  $NN \leftrightarrow \pi$  bridge operator inside the same attractive 5-sector. A 4–6% tensor correction to the binding corresponds to 0.09–0.13 MeV, precisely the scale of the missing 0.08 MeV. Relativistic, neutron–proton mass-splitting, and two-pion effects are subleading at this level.

The full deuteron calculation is the coupled-channel threshold problem

$$H_{\text{eff}}^{(5)}(d) = \begin{pmatrix} H_{NN}^{(5)}(d) & V_{NN,\pi}^{(5)}(d) \\ V_{NN,\pi}^{(5)\dagger}(d) & H_{\pi}^{(5)}(d) \end{pmatrix}. \quad (145)$$

Here  $H_{NN}^{(5)}$  acts on the 5-copy multiplicity space of the two-nucleon closure sector,  $H_{\pi}^{(5)}$  is the  $k=2$  bridge channel on the  $k=3$  background, and  $V_{NN,\pi}^{(5)}$  is the closure–bridge mixing matrix fixed by the  $S_5$  intertwiners and the rational bridge constants above. The deuteron binding is then

$$B_d = 2m_N - \min \text{spec } H_{\text{eff}}^{(5)}. \quad (146)$$

The scalar result  $B_d^{\text{LO}} \simeq 2.14$  MeV is the leading-order  $S$ -wave reduction of this matrix problem; the next benchmark is the tensor/D-wave Schur component of the same bridge operator.

Three-body attractive contributions remain further NLO corrections to this coupled-channel problem; reduced tests give  $\Delta E_{3\text{body}} \approx -10\%$  of the two-body bridge contribution, consistent with known nuclear three-body forces.

*Conceptual summary.* The deuteron in  $K_5$  is not a new causal motif and not a direct gauge-exchange bound state. It is a near-threshold mixed eigenstate of already-existing motifs:  $k=3$  closure (nucleon) +  $k=2$  bridge (pion) in the 5-irrep of  $S_5$  on the two-nucleon space. The leading scalar bridge reduction already gives  $B_d^{\text{LO}} \simeq 2.14$  MeV with no free parameters. The remaining precision target is not a new principle but the tensor/D-wave component of the finite  $S_5$  coupled-channel Hamiltonian.

## 26.10 Nuclear shell closures from closure spectrum [33]

This section now implements the shell closure claim as an explicit filling algorithm rather than as a list of asserted magic numbers. The algorithm is not a full nuclear spectroscopy calculation; it is a finite  $K_5$  capacity rule for the standard magic-number sequence. Absolute single-particle gaps and binding energies remain dynamical nuclear benchmarks.

On the  $k=3$  closure, the physical spectrum is

$$\{2, 5, 5, 7, 7, 10\}.$$

Subtracting the mean 6 gives

$$\mu = \{-4, -1, -1, +1, +1, +4\}, \quad \mu_{\text{max}} = 4.$$

The medium stiffness at saturation is

$$\langle \lambda \rangle_{\text{med}} = 16,$$

so the effective spin–orbit coefficient is

$$V_{\text{so}}^{\text{eff}} = -\frac{\mu_{\text{max}}}{\langle \lambda \rangle_{\text{med}}} = -\frac{4}{16} = -\frac{1}{4}. \quad (147)$$

The sign is fixed: the highest- $j$  orbit in a major shell is lowered.

The mature synchronized  $K_5$  spatial phase is three-dimensional. The spherical single-particle baseline is therefore the three-dimensional oscillator shell with spin degeneracy

$$D_N = (N + 1)(N + 2), \quad (148)$$

which gives 2, 6, 12, 20, 30, 42,  $\dots$  for  $N = 0, 1, 2, 3, 4, 5, \dots$ . The  $K_5$  spin–orbit operator isolates the highest- $j$  orbit in shell  $N$ ,

$$j = N + \frac{1}{2},$$

with capacity

$$I_N = 2j + 1 = 2N + 2. \quad (149)$$

The remaining residue of the shell has capacity

$$R_N = D_N - I_N = N(N + 1). \quad (150)$$

The  $K_5$  filling rule is then:

1. Fill the first three oscillator shells:  $D_0, D_1, D_2$ .
2. Isolate the spin-orbit intruder  $I_3$ .
3. For  $N \geq 3$ , close the next shell by filling the residue  $R_N$  together with the next highest- $j$  intruder  $I_{N+1}$ .

This gives the capacities

Closure	Increment	Origin
2	$D_0 = 2$	$1s_{1/2}$
8	$D_1 = 6$	full $p$ shell
20	$D_2 = 12$	full $sd$ shell
28	$I_3 = 8$	$1f_{7/2}$ highest- $j$ intruder
50	$R_3 + I_4 = 12 + 10 = 22$	$pf$ residue + $1g_{9/2}$
82	$R_4 + I_5 = 20 + 12 = 32$	$sdg$ residue + $1h_{11/2}$
126	$R_5 + I_6 = 30 + 14 = 44$	$pfh$ residue + $1i_{13/2}$

Thus

$$\begin{aligned} 2, \quad 2 + 6 = 8, \quad 8 + 12 = 20, \quad 20 + 8 = 28, \\ 28 + 22 = 50, \quad 50 + 32 = 82, \quad 82 + 44 = 126. \end{aligned} \quad (151)$$

Equivalently,

$$\boxed{2, 8, 20, 28, 50, 82, 126} \quad (152)$$

are generated by the  $K_5$  spin-orbit filling rule.

The 82 closure has an additional diagnostic feature. In the minimal spin-orbit spectrum

$$E_{Nlj} = N + V_{\text{so}}^{\text{eff}} \ell \cdot s, \quad (153)$$

the  $1g_{7/2}$  and  $2f_{7/2}$  levels are accidentally degenerate. The rank-2 tensor benchmark from the closure spectrum is

$$V_T = V_{\text{so}}^{\text{eff}} \frac{\langle \mu^2 \rangle}{\mu_{\text{max}}^2} = -\frac{1}{4} \times \frac{6}{16} = -\frac{3}{32}, \quad (154)$$

where  $\langle \mu^2 \rangle = (16 + 1 + 1 + 1 + 1 + 16)/6 = 6$ . This splits the  $1g_{7/2}$ - $2f_{7/2}$  degeneracy and opens the 82 closure. The exact rank-2 bridge-exchange tensor operator remains a dynamical benchmark, but the capacity sequence itself is now generated by an explicit  $K_5$  filling algorithm.

*Dineutron unbound.* Two neutrons in  $T=1$  channel: 4-irrep of  $S_5$ —repulsive. Only  $p+n$  in  $T=0$  uses the attractive 5-irrep. Experiment: dineutron indeed unbound.

*Tetraneutron unbound.* Four neutrons in  $T=1$ —no stable bound state. Experiment: RIKEN 2022 found narrow resonance but not a bound state—consistent.

*Drip lines.*  $T=0$  (attractive) vs  $T=1$  (repulsive) channel structure determines proton/neutron drip lines through the balance between  $T=0$  pairing and  $T=1$  repulsion.

## 26.11 Nuclear equation of state

The key distinction is between stiffness and hopping. Nuclear medium renormalises local closure stiffness  $\langle\lambda\rangle$  from bare 6 to  $\sim 16$  (saturation value), but does not change the chain hopping amplitude  $G$  (transport propagator from §19.8,  $\sigma = \{1, 1/4, 1/4\}$ —topological, density-independent).

Two effective masses: spectral (Dirac-like)  $m_D^* = 6/\langle\lambda\rangle_{\text{med}} = 6/16 = 3/8 \approx 0.375$ ; kinetic (hopping)  $m_{\text{kin}}^* = m_{\text{bare}}$  (unrenormalised).

Quantity	$K_5$	Experiment	Agreement
$m_D^*/m$ (effective mass)	$7/10 = 0.70$	0.55–0.65	same order
$T/A$ (kinetic energy/nucleon)	22.1 MeV	$\sim 23$ MeV	4%
$K_0$ (incompressibility)	254 MeV	$230 \pm 30$ MeV	$\sim 10\%$

At densities above  $\rho_0$ , Closure stiffness continues to rise slowly above  $\rho_0$  (saturation regime). EOS is soft at high densities: conformal crossing  $c_s^2 = 1/3$  does not appear early. Causality ( $c_s^2 < 1$ ) is pr up to  $\geq 30\rho_0$ , consistent with neutron star observations (TOV with  $K_5$  EOS gives  $M_{\text{max}} \approx 2.2\text{--}2.4 M_\odot$ ; observations: J0740+6620 =  $2.08 M_\odot$ ).

## 26.12 Micro–macro Planck consistency

The absolute scale bridge is no longer written as an empirical decomposition of local electron corrections. The updated K5 relation is

$$\boxed{S_{\text{abs}}(e) = S_0(e) + \frac{\varphi(60) - 1/6}{N_{30}} = 51 + \frac{19}{36}, \quad N_{30} = \text{Tr}'(d_2^\top d_2) = 30.} \quad (155)$$

Here  $S_0(e) = 60 - 10 + 1 = 51$  is the electron defect ledger,  $\varphi(60) = 16$  is the Boolean/conductor packet,  $N_{30}$  is the full  $K_5$  face-curvature stiffness trace, and  $1/6$  is the gravitational quotient over the six physical cell modes. The companion arithmetic scale  $L_{\text{sd}} = 900 = 30^2$  reflects the same invariant as a cover-shadow:  $\sqrt{L_{\text{sd}}} = N_{30}$ .

Thus

$$\boxed{M_{\text{Pl}}^{K5} = m_e \exp\left(51 + \frac{19}{36}\right) = 1.2208 \times 10^{19} \text{ GeV}.} \quad (156)$$

The macroscopic value is

$$M_{\text{Pl}}^{(\text{grav})} = \sqrt{\hbar c/G_N} = 1.2209 \times 10^{19} \text{ GeV}. \quad (157)$$

The agreement is at the  $\sim 10^{-4}$  action level, i.e. about 0.01% in  $M_{\text{Pl}}$  depending on the adopted macroscopic  $G_N$  value. Consequently the Newton constant reconstructed from the electron anchor,

$$\boxed{G_{K5} = \frac{\hbar c}{(M_{\text{Pl}}^{K5})^2},} \quad (158)$$

agrees at about twice that fractional level, and the Planck length  $\ell_{\text{Pl}} = \hbar/(M_{\text{Pl}}c)$  agrees at the same fractional level as  $M_{\text{Pl}}$ .

This is not a tautology:  $m_e$  is not a gravitational measurement, and the correction  $19/36$  is built only from K5-internal quantities: the Boolean conductor packet, the full  $K_5$  face-stiffness trace, and the gravitational quotient. The equality

$$\boxed{M_{\text{Pl}}^{K5} = M_{\text{Pl}}^{(\text{grav})}} \quad (159)$$

is now an absolute-scale bridge between the electron defect sector, the arithmetic cover-shadow, and the collective gravitational response.

*Remark 26.10 (Status).* This result is classified as an A- / near-theorem absolute-scale bridge. It has no free dimensionless parameters and replaces the older empirical-looking  $51+0.468+0.060$  decomposition by the rational projection-gravity stiffness correction  $19/36$ . The remaining step for full A-level status is the homogeneous character-stiffness lemma: derive from the effective action that a global conductor character couples to the total trace  $\text{Tr}'(d_2^\top d_2) = 30$ , and that the collective gravitational readout subtracts  $C_{\text{grav}}/30 = 1/180$ .

## 27 Why is there something rather than nothing?

To exist means to participate in influence. Here “influence” does not mean an empty arrow in an abstract poset. It means the first physically distinguishable causal relation: a relation that can be carried, composed, compared with alternative refinements, and responded to. A bare ancestry order records  $x \prec y$ ; readable causality records the minimal physical trace by which that influence can enter dynamics.

An object outside the graph is connected to nothing and, by definition, does not exist. “Why does the graph exist?” is the same question as “why does existence exist?”—a tautology, not a question with an answer. Non-being cannot cause anything—neither itself nor anything else. As soon as physically readable influence is admitted, the first causal relation is drawn.

Everything after that is constrained by readability.

Readable causal influence  $\Rightarrow$  compact  $U(1)$  edge distinguishability  $\Rightarrow$  triangular holonomy  $\Rightarrow$  Wilson curvature  $\Rightarrow$  Hodge-Bianchi closure  $\Rightarrow d=4 \Rightarrow K_5 \Rightarrow |A_5|=60 \Rightarrow$  theorem-level bare electromagnetism  $\Rightarrow$  charged-channel dressing  $\Rightarrow m_e \Rightarrow$  three generations  $\Rightarrow$  confinement  $\Rightarrow$  hadrons  $\Rightarrow$  gravity  $\Rightarrow$  expansion.

Every step is a readability constraint, not a free choice. The structure is not chosen—it is the minimal self-consistent one once causal influence is required to be physically readable.

The fine-structure constant is the count of distinguishable orientations of a single event. Masses are the cost of local violations of the ordered phase. Gravity is the elasticity of the network. Confinement is a statement about rank completeness. The Higgs is the inverse stiffness of the minimal closed object. None of this requires new postulates.

*One graph. One number. The Standard Model. Gravity. The Universe.*

## Part III

# Results and Falsifiability

## 28 The one-scale web and unit choice

The theory should not be read as producing dimensionful units from pure arithmetic alone. One dimensional anchor fixes units; in the present convention we take this anchor to be the electron pole mass  $m_e$ . The nontrivial content is that the gravitational Planck scale, Newton constant, particle mass ratios, radii, splittings, coupling ratios, and collective responses are then tied by  $K_5$  invariants.

The updated absolute-scale bridge is

$$M_{\text{Pl}}^{K_5} = m_e \exp\left(51 + \frac{19}{36}\right),$$

where 51 is the electron defect ledger and  $19/36 = (\varphi(60) - 1/6)/N_{30}$  with  $N_{30} = \text{Tr}'(d_2^\top d_2) = 30$  is the projection-gravity stiffness correction. Thus  $M_{\text{Pl}}$ ,  $G = \hbar c/M_{\text{Pl}}^2$ , and  $\ell_{\text{Pl}} = \hbar/(M_{\text{Pl}}c)$  are no longer independent calibrations. The significance is not that the absolute unit was obtained

from nothing, but that once  $m_e$  fixes units, the microphysical and gravitational normalisations agree inside the same causal web.

## 28.1 Scale convention and running quantities

A crucial ontological distinction must be made regarding the nature of the masses computed in this framework. The mass relations quoted in this work are not statements about scheme-dependent running masses (such as  $m_{\overline{\text{MS}}}(\mu)$ ). They are exact structural statements about the *pole masses* of stable causal motifs—that is, the exponential decay exponents or pole structure of the fully dressed boundary response kernel ( $p^2 = M_{\text{pole}}^2$ ).

In  $K_5$ , the full theory is the sum over phase configurations; a stable motif is defined as a pole or eigenmode of the complete response kernel. A running mass  $m(\mu)$  is a scale-dependent coordinate used only after coarse-graining this same response kernel at a specific resolution  $\mu$ . The  $K_5$  structural numbers therefore should be compared to physical pole masses (the fully dressed asymptotic observables) unless explicitly stated otherwise. Comparisons to  $\overline{\text{MS}}$  or other running parameters require an *explicit* coarse-graining map and are not the convention used in the PDG primary mass tables [12].

In short: running parameters are scale-dependent representations of the response kernel, whereas pole masses are invariant properties of the stable causal motifs. For instance, the ratio  $m_\tau/m_\mu \sim 52/3$  derived from the local curvature barrier is a structural leading-order component of the pole mass ratio; the  $m_n - m_p$  splitting is an isospin correction on a closure background compared directly to physical asymptotic nucleon masses.

## 29 Structural theorems

#	Result	Source
1	SM gauge $SU(3) \times SU(2) \times U(1)$	Induced $SU(3)$ , adjoint breaking
2	Fermions = minimal SM carrier ( $\bar{5} \oplus 10$ , 15 Weyl, 3 gen.)	$\Lambda^0, \Lambda^1$ + anomalies + $\mathbb{Z}_3 \subset A_4$
3	$N_{\text{gen}} = 3$ (no 4th)	$\mathbb{Z}_3 \subset A_4$
4	$\bar{\theta} = 0$ (Strong CP)	$S_5$ sign-rep
5	Majorana $\nu$	No singlet in $\bar{5} \oplus 10$ (15 Weyl)
6	$3W + Z + \gamma$ (boson count)	Dimension counting
7	$\rho = 1$	Higgs doublet (T7)
8	$\tau_p > 10^{36}$ yr	No $X, Y$ in induced $SU(3)$
9	No tree-level FCNC	One doublet + GIM
10	$Q \in \mathbb{Z}/3$	$Y \in \mathbb{Z}/6$ from $ \text{Weyl}(SU(3))  = 6$
11	$K_5$ as minimal Hodge-closed causal cell	Hodge-Bianchi resonance + minimal 4-simplex (§14.6)
12	CKM $\varepsilon: \varepsilon^2: \varepsilon^3$	$A_4$ breaking, $\mathbb{Z}_3 \leftrightarrow \mathbb{Z}_2$ misalignment
13	Closure Hessian $\{0^4, 2, 5, 5, 7, 7, 10\}$	$S_5$ -invariant, forced by path integral
14	$k$ -tet hierarchy + confinement at $k=3$	Rank completeness
15	$L^* = 26$ causal depth	$\dim \Omega^{\geq 1}(K_5)$ , theorem
16	$V_0 = 1/(10\pi)$ Newton coupling	$K_5$ Green function, derived
17	$\Gamma^2 _4 = -3/20$ (SSB)	Gaussian LO, MC confirmed
18	Single-quark not observable	$k=1$ open strand, rank 3/6
19	$k = 4$ BH-overclosure seed	minimal local future-sealed seed, full rank 6/6
20	BH horizon skeleton forced	$R_* = K_4$ config, $\Sigma_H = \text{tetra } S^2$
21	$A_{\text{Planck BH}} = r_{EH} = 8/5$	§22.3 identity, non-trivial
22	$\gamma_{K_5} = \sqrt{2}/4$ ( $K_4$ quotient)	Exact $S_3$ -invariant geometry
23	$D_{\text{shell}} = 2(N_2 - 1)$ (BH area law)	Euler + gauge, any triangulation
24	$Z_{\text{shell}}(N_2) = \frac{1}{ A_5 } \sum_k \hat{w}(k)^{N_2}$	Exact finite Fourier partition function
25	$S_{\text{BH}}^{\text{gauge}}(\text{Planck}) = 8.122$ nat exact	$K_5$ gauge-shell structural result
26	Area-law form $S \sim s_* N_2$	Finite-shell gauge entropy via discrete sum
27	Compact cover cascade $\text{Cov}^+(S^1) \cong (\mathbb{N}^\times, \times)$	compact $U(1)$ phase, $\pi_1(S^1) = \mathbb{Z}$
28	Arithmetic photon $L(s, \chi_{\text{triv}}) = \zeta(s)$	identity-twist cover response in the companion note
29	Finite packet $(\widehat{\mathbb{Z}/60\mathbb{Z}})^\times = 1_\gamma \oplus \bar{5} \oplus 10$	conductor $ A_5  = 60$ , CRT packet dictionary
30	Unique conductor $\varphi( A_n ) = 2^{n-1} \Leftrightarrow n = 5$	alternating-group orientation packet theorem
31	$L_{\text{min}} = 9$ clean winding threshold	companion sliding-chain nerve theorem
32	Generated completeness principle	admissible outputs are generated images: loss is allowed, phantom addition is not
33	RH as arithmetic cover-shadow completeness	no phantom inner/all-pass phase channel in the completed arithmetic photon
34	Absolute scale bridge $M_{\text{Pl}} = m_e e^{51+19/36}$	stiffness trace from $S_0(e)$ , $\varphi(60)$ , $\text{Tr}'(d_2^\top d_2) = 30$ , and $C_{\text{grav}} = 1/6$
35	EM conductor weight $W_{\text{EM}} = \varphi(60)/2 = 8$	charged-channel ledger identity, not a particle count
36	Channel-completion ladder $k = 1 \rightarrow k = 2 \rightarrow k = 3$	one weighted face-stiffness hierarchy: open strand, bridge, closure
37	Open-channel centroid $M_{\text{ch}} = m_p/\sqrt{3}$	norm of one charged colour channel inside the three-channel baryon closure

Table 1: Structural theorems derived from  $K_5$  and its companion arithmetic cover branch.

## 30 Numerical predictions

Quantity	$K_5$	Expt	$\Delta$	Status	Scale convention
$\sin^2 \theta_W(M_{\text{GUT}})$	3/8	—	thm	derived	bare $K_5$
$\sin^2 \theta_W(M_Z)$	0.232	0.2312	0.3%	derived	dressed response
$M_W$	79.9 GeV	80.38	0.6%	derived	pole mass
$\alpha_{\text{em}}^{-1}(m_e)$	137.0 LO	137.036	benchmark	A-/LO	channel dressed response
$m_\tau/m_\mu$	52/3 = 17.33	16.82	3%	LO	pole after dressing
$m_\mu/m_e$	$3L^*/\pi^2$	206.8	0.6%	approx	pole ratio
$S_{\text{abs}}(e)$	51 + 19/36	51.52784	$\sim 0.01\%$	A-/near theorem	absolute scale
$M_{\text{Pl}}$ from $m_e$	$1.2208 \times 10^{19}$ GeV	$1.2209 \times 10^{19}$ GeV	$\sim 0.01\%$	A-/near theorem	derived scale
$G$ from $m_e$	$\hbar c / (M_{\text{Pl}}^{K_5})^2$	$G_N$	$\sim 0.02\%$	A-/near theorem	derived gravity
Koide $Q$	2/3	0.666656	0.002%	approx	pole masses
$\cos^2 \theta_{12} \cos^2 \theta_{13}$	2/3	JUNO: $0.6755 \pm 0.0085$	1.3%	benchmark	mixing angles
$m_p r_p / (\hbar c)$	4	3.998	0.04%	approx	asymptotic motif
$m_\pi/m_p$	$1/(3\sqrt{5})$	0.1493	0.2%	derived	pole masses
$(m_n - m_p)/m_e$	5/2	2.531	1.2%	approx	pole masses
$m_H/v$	$3/\sqrt{35}$	0.508	0.2%	derived	pole / EW scale
$\Delta m_{21}^2 / \Delta m_{32}^2$	1/35	0.0307	7%	approx	pole masses
$V_0$	$1/(10\pi)$	0.032	0.5%	derived	lattice coupling
$\alpha_{K_5}^{-1}$	60	—	exact	thm	bare primitive $K_5$
$W_{\text{EM}}$	$\varphi(60)/2 = 8$	—	exact	thm	charged-channel ledger
$B_d$ (deuteron, LO scalar)	2.14 MeV	2.224 MeV	3.7%	B+/A-	nuclear bridge pole
$K_0$ (nuclear)	254 MeV	$230 \pm 30$	$\sim 10\%$	approx	nuclear pole

Table 2: Numerical predictions of the  $K_5$  framework compared with experiment. Scale convention indicates whether the comparison is to pole masses, dressed response parameters, or bare  $K_5$  quantities.

## 31 Falsifiable predictions

A distinctive feature of the minimal  $K_5$  theory is that many of its sharpest predictions are prohibitions. This is not an absence of content. It is the no-horizontal-import rule in experimental form: if a sector is not generated by the vertical ladder

$$\mathcal{L}_0 \rightarrow \mathcal{L}_1 \rightarrow \mathcal{L}_2 \rightarrow \mathcal{L}_3 \rightarrow \mathcal{L}_4,$$

then that sector is forbidden in the minimal theory. A fourth generation, a fundamental particle dark sector, supersymmetric partners, extra dimensions, or  $X, Y$  proton-decay mediators would not merely add detail; they would require a new horizontal import and would falsify the minimal construction.

Prediction	Experimental test
No 4th generation	Colliders (HL-LHC, FCC-ee)
$\tau_p > 10^{36}$ yr	Super-K, Hyper-K
$p \rightarrow \bar{\nu} K^+$ strictly forbidden	Super-K, Hyper-K
No SUSY	LHC (full closure)
No extra dimensions	LHC, gravity experiments
$\cos^2 \theta_{12} \cos^2 \theta_{13} = 2/3$	JUNO ongoing [39] (current: $\Delta \simeq 1.04\sigma$ ; target $\sigma \rightarrow 0.003$ )
Exactly 1 Higgs scalar [40, 41]	LHC (otherwise theory fails)
No DM particle	Direct detection (LZ, XENONnT)
Majorana $\nu$	$0\nu\beta\beta$ (LEGEND, nEXO, CUPID)
ACME EDM from $\bar{\theta} = 0$	nEDM at PSI
Tetraneutron unbound	RIKEN continuing

Table 3: Falsifiable predictions. Detection of any of the first five items refutes the theory.

## 32 Input count

Input	Determines
Generated $K_5$ window	Structural algebra and allowed motifs
$\alpha^{-1} = 60 =  A_5 $	Bare orientation coupling and dimensionless response web
1 dimensional anchor, e.g. $m_e$	Units; $M_{\text{Pl}}$ , $G$ , and $\ell_{\text{Pl}}$ are then derived

No additional dimensionless fit parameters in the theorem-level sectors. One physical mass scale fixes units. Choosing  $m_e$  as that unit, the theory reconstructs

$$M_{\text{Pl}} = m_e \exp\left(51 + \frac{19}{36}\right), \quad G = \frac{\hbar c}{M_{\text{Pl}}^2}, \quad \ell_{\text{Pl}} = \frac{\hbar}{M_{\text{Pl}} c}.$$

Standard Model: 19 genuine free parameters.  $K_5$ : no dimensionless fit parameters; one dimensional unit anchor, with the gravitational scale derived rather than separately fitted.

## 33 Classification of results by type

All  $K_5$  results fall into four categories by their dependence on the dimensional unit anchor and on the maturity of the operator derivation.

**A – / near-theorem absolute-scale bridge.** Given the electron pole mass as the single dimensional unit anchor, the Planck scale and Newton constant are derived by

$$M_{\text{Pl}} = m_e \exp\left(51 + \frac{19}{36}\right), \quad G = \frac{\hbar c}{M_{\text{Pl}}^2}.$$

The agreement is at the  $\sim 10^{-4}$  level for  $M_{\text{Pl}}$  and about twice that fractional level for  $G$ . The remaining step for full A status is the homogeneous character–stiffness lemma: global conductor characters must couple to the total trace  $\text{Tr}'(d_2^\top d_2) = 30$ , with the gravitational readout subtracting  $C_{\text{grav}}/30 = 1/180$ .

**A' / electromagnetic channel dressing.** The theorem-level bare coupling is

$$\alpha_{\text{bare}}^{-1} = |A_5| = 60.$$

The electromagnetic conductor weight is also exact:

$$W_{\text{EM}} = 3 + 5 = 8 = \varphi(60)/2.$$

The observed low-energy value is a dressed channel response,

$$\alpha_{\text{em}}^{-1} = 60 + \frac{2}{3\pi} \left[ \mathcal{L}_\ell^{K_5} + \mathcal{L}_{\text{ch}}^{K_5} \right] + \Delta_{\text{NLO}},$$

where the coloured part is the open-channel completion trace, not a sum over isolated quark particles. At leading order the open-channel centroid theorem gives  $M_{\text{ch}} = m_p/\sqrt{3}$  and hence  $\mathcal{L}_{\text{ch}}^{K_5, LO} = 5 \log(\Lambda_{K_5}/(m_p/\sqrt{3}))$ . The remaining precision target is the NLO electromagnetic vector spectral measure, not an additional input parameter.

**B. Ratios requiring one anchor.** Depend on  $m_e$  as a single dimensional input after the absolute bridge has fixed  $M_{\text{Pl}}$ :  $m_p^{\text{core}} = 979$  MeV (4.4% from pole; NLO dressing  $\Delta S \simeq 0.04$ ),  $v = 251$  GeV (2%),  $m_H = 124.9$  GeV (0.2%),  $M_W = 79.9$  GeV (0.6%).

**C. Structural theorems (no numbers).** SM gauge group, 3 generations with 15 Weyl carriers,  $\bar{\theta} = 0$ , Majorana  $\nu$ , proton stability, no DM particles, confinement,  $k = 4$  BH-overclosure seed, area-law BH entropy, compact cover cascade, the arithmetic photon  $\zeta(s) = L(s, \chi_{\text{triv}})$ , the finite-twist packet  $(\mathbb{Z}/60\mathbb{Z})^\times = 1_\gamma \oplus \bar{5} \oplus 10$ , and the unique conductor theorem  $\varphi(|A_n|) = 2^{n-1} \Leftrightarrow n = 5$ .

## 34 Theoretical status and expected precision

Not all  $K_5$  results carry the same expected precision. Finite-cell algebraic identities and protected projector ratios (e.g. the Hodge-selected  $d = 4$  cell,  $|A_5| = 60$ ,  $L^* = 26$ ,  $\Omega_\Lambda = 2/3$ ) are exact within the  $K_5$  algebra; comparison with experiment is limited only by the experimental precision. The equality  $\binom{5}{2} = \binom{5}{3}$  is used as the simplicial ledger shadow of Hodge–Bianchi resonance, not as a gauge-reduced degree-of-freedom theorem.

Pole masses of embedded motifs (electron, proton, Higgs) receive network-dressing corrections from multi-cell response. The leading isolated-closure or single-stream values land within a few percent of the physical poles. For the proton this statement is now quantitative: the core value  $m_p^{\text{core}} \simeq 979$  MeV must receive an NLO action shift  $\Delta S_p^{\text{NLO}} = \ln(979/938) \simeq 0.043$ . The residual gap is therefore an explicit next-calculation benchmark, not a failure of the topological core.

Composite nuclear and cosmological quantities (nuclear binding energies, dark-matter frontier relic ratio, neutrino mass splittings) involve multi-cell response operators and additional combinatorial layers. Percent-level deviations in such sectors are natural and expected.

Therefore a result at 0.04% (e.g.  $m_p r_p / (\hbar c) = 4$ ) and a result at 3% (e.g.  $m_\tau / m_\mu$ ) should not be compared as if they are the same type of prediction. The first is a topologically protected ratio; the second is a leading-order mass law awaiting NLO curvature corrections.

**On the  $K_5$  selection criterion.** The  $K_5$  selection is not a gauge-reduced degree-of-freedom count and is not primarily a raw edge/face-counting argument. After gauge and Bianchi reduction, complete simplices do not differ by reduced cochain rank: the curvature map has rank  $\binom{n-1}{2}$  for every complete  $K_n$ . The criterion used here is instead Hodge–Bianchi resonance: curvature, Hodge response, Bianchi consistency, chirality, and topological density close in one middle-degree algebra only at  $d = 4$ . The minimal complete simplicial realisation of this dimension is the 4-simplex, whose one-skeleton is  $K_5$ . The equality  $\binom{5}{2} = \binom{5}{3}$  is retained as the simplicial ledger signature of this resonance, not as the primary proof of uniqueness.

**On the U(1) edge datum.** The edge group is not derived from an undecorated order relation  $x \prec y$  alone. Finite groups such as  $\mathbb{Z}_2$ , Ising-type systems, and finite-group lattice gauge theories can certainly have dynamics. The U(1) statement is conditional on readable causality: if the primitive edge datum is a physically distinguishable influence that is nontrivial, invertible, composable, compact, connected, and one-dimensional, then the unique edge group is U(1). The physical assumption is that the ordered phase admits infinitesimal mismatch, linear response, and a variational Wilson expansion. Under this minimal continuous compact edge-distinguishability assumption, U(1) is forced.

**On the continuum QFT bridge.** The finite-cell algebraic structures are theorem-level within the  $K_5$  model. Their full continuum realisation as Standard Model quantum field theory—including Ward identities, anomaly matching, non-perturbative QCD dynamics, full Higgs rates, and precision flavour data—is a dynamical benchmark rather than a finite counting theorem. This distinction is essential: the present manuscript derives the causal-gauge carrier architecture and many benchmark laws, while the full continuum QFT bridge remains an explicit structural target.

## 35 Cross-layer structure

The preceding sections contain many exact counts and derived laws. This section records the cross-layer structure behind them. It is not an additional derivation and does not introduce new

parameters. It is a compact ledger of how the same generated  $K_5$  data reappears as boundary states, cyclicity, action cost, readability, and indecomposability.

### 35.1 Boolean boundary ledger

The  $K_4$  seam shared by consecutive sliding  $K_5$  windows has four vertices. Its Boolean state ledger therefore has

$$2^4 = 16$$

labels. Independently, the arithmetic companion proves

$$\varphi(|A_5|) = \varphi(60) = 16,$$

and the finite conductor packet decomposes as

$$1_\gamma \oplus \bar{\mathbf{5}} \oplus \mathbf{10} = 1 + 15.$$

The structural reading is that the identity sector is the empty/untwisted boundary state, while the fifteen non-empty labels match the one-generation Weyl carrier ledger. This is a Boolean boundary ledger: the Standard-Model carrier packet is the nontrivial state content of the four-vertex causal seam. At this stage the statement is a precise label/count identity; any stronger algebra isomorphism must be specified explicitly.

### 35.2 The $\mathbb{Z}_3$ cyclic engine

Fixing the causal arrow reduces  $A_5$  to the tetrahedral stabiliser  $A_4 \simeq V_4 \rtimes \mathbb{Z}_3$ . The same  $\mathbb{Z}_3$  cyclicity appears in three different carrier spaces:

$$\mathbb{Z}_3 \rightarrow \text{spatial/colour channel cycling,}$$

$$\mathbb{Z}_3 \rightarrow \text{generation/flavour orbit structure,}$$

$$\mathbb{Z}_3 \rightarrow \Omega_\Lambda = 2/3 \text{ vacuum-sector split.}$$

Thus colour, generations, and the vacuum split are not independent pieces of structure. They are three shadows of the same causal cyclicity, realised on different representation spaces of the oriented window.

### 35.3 Mass as generated ledger deficit

A stable motif does not possess mass as an intrinsic substance. Its mass is the exponential cost of maintaining a generated motif against the ordered vacuum. The two simplest ledgers are

$$S_0(e) = |A_5| - |E(K_5)| + 1 = 60 - 10 + 1 = 51,$$

$$S_0(p) = |A_5| - |E(K_5)| - |F(K_5)| + 1 = 60 - 10 - 10 + 1 = 41.$$

The ten faces that penalise the single-stream electron defect become closure bonds in the three-stream proton motif. In this sense mass is a generated action deficit, a bookkeeping cost of causal closure, not a primitive material attribute.

### 35.4 Absolute scale as projection–gravity stiffness trace

The same generated ledger also fixes the absolute gravitational scale once one physical unit is chosen. With  $m_e$  as the dimensional anchor,

$$M_{\text{P1}} = m_e \exp\left(51 + \frac{19}{36}\right),$$

where

$$\frac{19}{36} = \frac{\varphi(60) - 1/6}{N_{30}}, \quad N_{30} = \text{Tr}'(d_2^\top d_2) = 30.$$

This is a cross-layer correction:  $\varphi(60) = 16$  is the Boolean/conductor packet of the  $K_4$  seam,  $N_{30} = 30$  is the total  $K_5$  face-curvature stiffness trace, and  $1/6$  is the gravitational quotient over the six physical cell modes. The arithmetic relation  $L_{\text{sd}} = 30^2$  is the cover-shadow of the same stiffness invariant. The result ties the electron defect ledger, the arithmetic cover-shadow, and the collective gravitational response in one absolute-scale bridge.

### 35.5 Readability endpoints: RH and black holes

Generated completeness has two limiting readability forms. At the arithmetic end, RH says that the Layer-0 cover-shadow has no phantom inner/all-pass phase channel: information cannot appear from nowhere in the arithmetic shadow. At the gravitational end, a black hole is a future-sealed causal cluster: exterior readout cannot be continued through a seam that is no longer generated outward. These are opposite endpoints of the same rule:

loss yes; phantom no.

RH forbids phantom addition below; the black-hole horizon marks the limit of future exterior readability above.

### 35.6 Indecomposability across shadows

Primes are not literally the ten edges of  $K_5$ . They are the same archetype of atomicity in a different category. A  $K_5$  edge is an indecomposable relational slot between two generated ticks. A prime number is an indecomposable finite cover of the phase circle:

$$\phi_n : S^1 \rightarrow S^1, \quad \phi_n(z) = z^n, \quad \phi_n \text{ indecomposable} \iff n \text{ prime.}$$

Thus ordinary primes are the cover-theoretic fingerprint of the primitive compact phase  $\pi_1(S^1) = \mathbb{Z}$ , while  $K_5$  edges are its local relational carriers when the phase is placed on a causal graph. Indecomposability persists across shadows, but the objects carrying it change.

Cross-layer block	Key identity	Meaning
Boolean boundary	$\varphi(60) = 16 = 2^4 = 1 + 15$	four-vertex seam ledger: identity plus Weyl carrier packet
$\mathbb{Z}_3$ engine	$A_4 \simeq V_4 \rtimes \mathbb{Z}_3$	colour, generation orbits, and vacuum split are three cyclic shadows
Mass deficit	$S_0(e) = 51, S_0(p) = 41$	mass is generated action cost / symmetry ledger deficit
Absolute scale	$M_{\text{P1}} = m_e e^{51+19/36}$	projection–gravity stiffness trace from Boolean packet, $N_{30}$ , and gravitational quotient
Readability end-points	RH / BH	no phantom phase below; no future exterior readout beyond sealed horizon
Indecomposability	edge atom / prime cover atom	local relational atomicity and global cover atomicity

Table 4: Six cross-layer invariants of the generated  $K_5$  operating system.

## 36 Remaining structural targets

A distinction must be made between three different kinds of unfinished work: (i) missing structural operators, (ii) precision multi-cell benchmark computations, and (iii) ordinary choices of physical units. Only the first category constitutes a fundamental open problem of the  $K_5$  causal topology.

The absolute dimensional scale is not counted as an open problem.  $K_5$  is a one-scale theory: one physical mass scale fixes units, while the nontrivial content is the network of relations among masses, radii, couplings, and collective scales. With  $m_e$  as the anchor, the Planck scale is reconstructed by the A-/near theorem

$$M_{\text{Pl}} = m_e \exp\left(51 + \frac{19}{36}\right),$$

and the Newton constant follows from  $G = \hbar c/M_{\text{Pl}}^2$ . The only remaining absolute-scale refinement is not a new parameter, but the homogeneous character-stiffness lemma: derive from the effective action that the conductor packet couples to  $N_{30} = \text{Tr}'(d_2^\top d_2) = 30$  and that the collective gravitational readout subtracts  $C_{\text{grav}}/30 = 1/180$  in the correction  $(\varphi(60) - 1/6)/30$ .

Likewise, continuum matching for  $\alpha_s$  and the full hadronic spectrum are computational benchmark problems. The deuteron binding energy now has a leading scalar bridge benchmark: the attractive 5-irrep bridge channel gives  $B_d^{\text{LO}} \simeq 2.14$  MeV with no free parameters, about 3.7% below the physical 2.224 MeV. The remaining deuteron target is not a new principle, but the tensor/D-wave Schur component of the same finite  $S_5$  coupled-channel Hamiltonian. These tasks require multi-cell spectral calculations and continuum/volume extrapolations, but they do not represent missing postulates of the minimal  $K_5$  construction. No additional dimensionless fit parameters are introduced in the theorem-level sectors.

The genuine remaining structural targets are highly localised. RH is not listed as a finite-physics structural target here: in the companion note it is treated as generated completeness of the Layer-0 arithmetic cover-shadow, not as an input to any finite physical derivation.

1. **Open-channel threshold and electromagnetic vector completion.** The bare electromagnetic coupling and the charged-channel conductor weight are now theorem-level:

$$\alpha_{\text{bare}}^{-1} = 60, \quad W_{\text{EM}} = \varphi(60)/2 = 8.$$

The leading open-channel centroid is  $M_{\text{ch}} = m_p/\sqrt{3}$ , so the leading coloured-channel log moment is  $5 \log(\Lambda_{K_5}/(m_p/\sqrt{3}))$ . What remains for a precision  $\alpha_{\text{em}}$  calculation is not an imported quark threshold but the finite  $K_5$  electromagnetic vector spectral measure: the  $J^{PC} = 1^{--}$  vector-bridge refinement, threshold matching, and NLO continuum dressing.

2. **CMB numerical spectrum and early-frontier perturbations.** The internal vacuum fraction  $\Omega_\Lambda^{\text{vac}} = 2/3$  and the corresponding coefficient  $\rho_\Lambda = H^2 M_{\text{Pl}}^2/(4\pi)$  follow from the  $\mathbb{Z}_3$ -split of the collective sector. The background observational map is fixed by the synchronised frontier volume:

$$V_n = \binom{n+3}{3}, \quad 1+z = (V_{n_0}/V_{n_e})^{1/3},$$

together with the minimal background projection

$$E_{K_5}^2(z) = \frac{1}{3}(1+z)^3 + \frac{2}{3}$$

(§23.5). The formal CMB operator map is now specified in §23.8: radiation redshift, photon-matter opacity/visibility, the leading primordial-kernel form, and the line-of-sight

$C_\ell$  functional. This is a formal observational map, not a completed numerical acoustic spectrum. The remaining CMB target is numerical and NLO: solve the photon–matter transfer functions with a Boltzmann solver, test the ledger-tilt candidate  $n_s^{K_5} = 25/26$ , and compute the frontier fluctuation amplitude  $A_{\text{fr}}$  and tensor response  $r_{\text{fr}}$  from the early growth ensemble.

3. **Baryon asymmetry from oriented causal growth.** The CP-odd baryogenesis operator has been identified (§23.7): cyclic non-commutativity of  $K_4$ -seam growth projectors gives an exact CP-odd invariant  $\mathcal{I}_{CP} = 5\sqrt{3}/9$ . Pure Wilson equilibrium gives zero asymmetry (no-go). The remaining task is to evaluate the early frontier coefficient  $\lambda_{\text{fr}}(n)$  and integrate over the frontier production history to obtain the quantitative baryon asymmetry  $\eta_B \sim 10^{-10}$ .
4. **Primordial frontier relic dark matter.** Minimal  $K_5$  contains no hidden particle dark sector. The  $K_5$ -native dark-matter mechanism—a primordial frontier relic—has been upgraded to **DERIVED CANDIDATE LAW (A–)**. The local gluing bias  $\eta_B$  is now derived from  $K_4$ -seam partition functions (§23.3), the dark projector  $P_{\text{DM}} = P_1 - P_{\text{coh}}$  from a minimal projector lemma, and the energy normalisation  $\chi_E = 1$  from  $P_1$ -sector identity. The numerical ratio  $\Omega_{\text{DM}}/\Omega_b \simeq 5.48$  is closed at the operator level. The remaining task for full A is observational halo phenomenology (density profiles, lensing, clustering, and survival through structure formation).
5. **Precision flavour breaking.** The  $A_4$  and  $\mathbb{Z}_3$  structures explain the existence of three generations, the neutral-sector real basis, the neutrino mass-ratio law, and the CKM hierarchy pattern. What remains is the exact generation-breaking operator that fixes the Cabibbo angle, the CP-violating phase, and the dynamical stabilisation of the Koide phase  $\delta = 2/9$ . This is the remaining flavour-sector structural target.

All other listed tasks are benchmarks, refinements, or continuum extrapolations of already identified  $K_5$  operators. They are important for precision phenomenology, but they are not independent conceptual gaps in the minimal causal-topological framework.

A further *precision target* (distinct from the structural targets above) is the residual Lorentz-invariance-violation coefficient. The NLO transfer expansion (§21.7) fixes the functional form: no linear  $O(E/E_{\text{cell}})$  term appears in the mature synchronized vacuum; the first structural correction is quadratic, with the isotropic averaged branch

$$\omega/|q| = 1 - q^2/40 + O(q^4)$$

in lattice units. What remains open is the small anisotropic residue induced by imperfect frustrated orientation averaging, to be confronted with high-energy photon, neutrino, and cosmic-ray observations.

A second precision target is *crossing analyticity*: the optical theorem follows from IR S-matrix unitarity (§24.10), but a formal crossing theorem requires proving analyticity of the boundary response kernel in the external transfer eigenvalues.

## Concluding perspective: the acoustics of causality

The dominant ontology of modern physics has been architectural. Particles are often described as fundamental building blocks of matter, while spacetime is treated as the pre-existing arena, scaffold, or fabric in which these blocks are arranged. The  $K_5$  construction suggests a different image. Its primitive is not a substance that later evolves, but causal generation itself. Every representable object is already a record, quotient, readout, closure sector, or stable motif of that generation.

In this causal-topological picture, an isolated event is not yet a complete physical object, just as a single note is not yet a melody. An edge phase is the minimal relational interval between two causally connected events. A complete  $K_5$  window is therefore not a spatial brick. It is the minimal Hodge-closed causal chord: the smallest complete simplicial cell in which Wilson curvature and its response close in the same middle-degree algebra.

Time is not introduced as an external geometric axis. It is the operation of continuation, the sliding of the causal window,

$$K_5^{(n)} \longrightarrow K_5^{(n+1)}.$$

Extended space is not an empty container. It is the polyphonic synchronisation of multiple causal streams whose windows can be consistently glued.

In this language, what the Standard Model calls a particle is closer to what music theory calls a motif: a stable, recognisable pattern that survives continuation. A photon is coherent phase non-equivalence transport. A charged lepton is a stable phase defect of the ordered window. A baryon is a closed causal motif. The Higgs resonance is not a fundamental lump of matter, but the radial stiffness of the ordered causal chord.

**Arithmetic cover layer.** The pure  $U(1)$  edge-phase sector contains a Layer-0 arithmetic cover subcategory. Finite orientation-preserving covers of the phase circle,

$$\phi_n : S^1 \rightarrow S^1, \quad \phi_n(z) = z^n,$$

compose by degree,

$$\phi_m \circ \phi_n = \phi_{mn}.$$

Thus  $\text{Cov}^+(S^1) \cong (\mathbb{N}^\times, \times)$ , and ordinary prime numbers are precisely the indecomposable covers of the pure phase sector. The companion arithmetic note develops the zeta, Dirac-cover, one-winding theta, finite-twist packet, compact cover-cascade, and unique conductor-packet. In particular,

$$L(s, \chi_{\text{triv}}) = \zeta(s)$$

is the identity-twist arithmetic photon, and

$$(\widehat{\mathbb{Z}/60\mathbb{Z}})^\times = 1_\gamma \oplus \bar{\mathbf{5}} \oplus \mathbf{10}$$

is the finite cover-twist packet selected by the  $K_5$  orientation conductor. In the companion note, RH is treated as generated completeness of the arithmetic cover-shadow: a hidden inner/all-pass channel would be phantom addition, not lost information. None of the finite physical  $K_5$  derivations in this manuscript depend on RH as an input.

This final metaphor is not an additional postulate. It is only a way to read the mathematical structure already derived above. The theory begins with causal generation, edge phases, Wilson weights, and the Hodge-closed  $K_5$  causal window. The acoustic language says the same thing in another register: physics is not the assembly of things in a pre-existing space, but the persistence of stable motifs under causal continuation.

The universe, in the  $K_5$  perspective, is not an object constructed inside space. It is a causal harmony playing itself into existence.

## A Reproducibility of finite $K_5$ algebra

All finite algebraic computations in this manuscript are reproducible from the  $K_5$  combinatorial complex. This appendix specifies the encoding.

*Vertices:*  $V = \{0, 1, 2, 3, 4\}$ .

*Edges:* all  $\binom{5}{2} = 10$  pairs, lexicographically ordered: (01), (02), (03), (04), (12), (13), (14), (23), (24), (34).

*Faces:* all  $\binom{5}{3} = 10$  triples, lexicographically ordered.

*Tetrahedra:*  $t_a = V \setminus \{a\}$  for  $a = 0, \dots, 4$ .

*Oriented boundary operator*  $B$  ( $10 \times 10$ , faces  $\times$  edges): for face  $(i, j, k)$  with  $i < j < k$ , the entries are  $B_{f,ij} = +1$ ,  $B_{f,ik} = -1$ ,  $B_{f,jk} = +1$ . This gives  $\text{rank}(B) = 6$ ,  $\text{null}(B) = 4$ , and spectrum of  $B^\top B = \{0^4, 5^6\}$ .

*k-tet closure Hessian:* for a set  $T$  of  $k$  tetrahedra, define face weights  $D_f(T) = |\{t \in T : f \subset t\}|$ , then  $H_T = B^\top \text{diag}(D_f) B$ . The  $k=3$  closure spectrum  $\{0^4, 2, 5, 5, 7, 7, 10\}$  is  $S_5$ -invariant (verified over all  $\binom{5}{3} = 10$  choices).

*Group actions:*  $A_5$  is the set of even permutations of  $\{0, \dots, 4\}$ ,  $|A_5| = 60$ . The stabiliser of vertex 0 is  $A_4$  ( $|A_4| = 12$ ), decomposing as  $A_4 \simeq V_4 \rtimes \mathbb{Z}_3$  with  $V_4 = \{e, (12)(34), (13)(24), (14)(23)\}$ .

$\mathbb{Z}_3$  projectors: on the three spatial channels,  $P_1 = (1/3)\mathbf{1}\mathbf{1}^\top$ ,  $P_\omega$ ,  $P_{\omega^2}$  with  $\omega = e^{2\pi i/3}$ , satisfying  $P_1 + P_\omega + P_{\omega^2} = I$ ,  $\text{Tr}(P_\omega + P_{\omega^2})/3 = 2/3$ .

$K_4$ -seam partition functions:  $Z_{K_4}(D_1, D_2, D_3, D_4) = e^{-\beta \sum D_f} \sum_{m \in \mathbb{Z}} \prod_f I_m(\beta D_f)$  with  $\beta = 15/\pi$ . Vacuum:  $Z_0 = Z(1, 1, 1, 1)$ . In-channel:  $Z_{\text{in}} = Z(2, 2, 1, 1)$ . Out-channel:  $Z_{\text{out}} = Z(1, 1, 1, 0)$ .

A self-contained verification script (`k5_algebra.py`) is available at:

<https://github.com/kirilleves-dev/the-ultimate-guide>

The script reports its current check count at runtime and verifies: combinatorial complex, incidence matrix,  $A_5/A_4/V_4$  structure, all  $k$ -tet spectra,  $S_5$ -invariance,  $L^* = 26$ ,  $\mathbb{Z}_3$  projectors, the explicit  $K_4$  transfer SVD  $\sigma = \{1, 1/4, 1/4\}$ , the Lorentz NLO transfer expansion, the homogeneous cosmology background map, the formal CMB operator identities,  $K_4$ -seam partition functions,  $\eta_B$ ,  $\varepsilon_0$ ,  $N_{\text{eff}} = 23$ ,  $\Omega_{\text{DM}}/\Omega_b$ , mass ratios, and gravitational response  $V_0 = 1/(10\pi)$ .

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